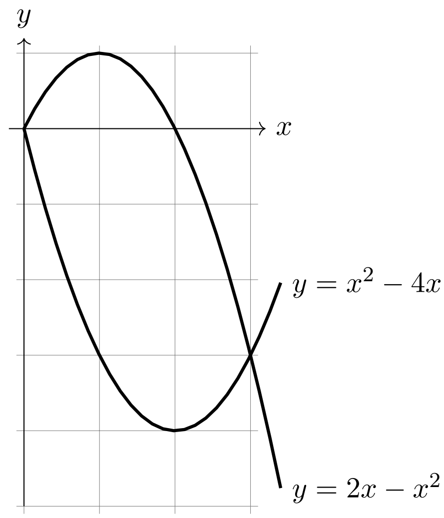
1. (34 points) Consider the region $\mathcal{R}$ between the two curves $y = x^2 - 4x$ and $y = 2x - x^2$ shown here:



   (a) Find the area of the region $\mathcal{R}$.
   (b) Find the x-coordinate of the centroid of this region.
   (c) Now, consider a new solid generated by rotating the region $\mathcal{R}$ around the line $y = 2$. Set up, but do not evaluate, an integral to calculate the volume of the solid.
   (d) Finally, consider the solid generated by rotating the region $\mathcal{R}$ around the line $x = -1$. Set up, but do not evaluate, an integral to calculate the volume of the solid.

2. (12 points) Solve the differential equation $\dfrac{dS}{dr} + \dfrac{2}{r}S = 0$ with initial value $S(2) = 3$. Assume $r > 0$ and $S > 0$. Simplify and express your answer in the form $S = S(r)$.

3. (24 points) The following problems are not related.
   (a) Find the length of the curve $g(y) = \frac{1}{3}y^3 + \frac{1}{4y}$ from $y = 1$ to $y = 2$.
   (b) Suppose you have a hemispherical bowl of radius 2 feet that is filled to a depth of 1 foot with water. What is the surface area of the bowl that is in contact with the water? Set up the integral but you need not evaluate it.

4. (18 points) Determine whether the sequences below converge or diverge. If a sequence converges, find the limit. Fully justify your work, and state any theorems that you use. For each sequence, you may assume that $n = 1, 2, 3, \ldots$
   (a) $a_n = \dfrac{\sin(n)}{n^{1/2}}$
   (b) $b_n = \dfrac{(2n+1)!}{(2n-1)!}$
   (c) $c_n = \ln(6n^2 + 7) - \ln(n^2 + 4)$

5. (12 points) Consider the sequence $a_n = \left(-\dfrac{3}{4}\right)^n$ for $n = 1, 2, 3, \ldots$
   (a) Find the value of s_2, the second partial sum.
   (b) Determine if the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges. If it converges, find its limit.

(c) Determine whether $\sum_{n=1}^{\infty}(a_n - 1)$ converges or diverges. If it converges, find its limit.