

1. (30 pts) The following three problems are not related.

(a) Beulah Bee travels along the path  $\mathbf{r}(t) = (6t)\mathbf{i} + (8 \cos t)\mathbf{j} + (8 \sin t)\mathbf{k}$ , starting at  $t = 0$ , and stops after covering a distance of  $15\pi$  units. What are the coordinates of the bee's position now? Simplify your answer.

(b) The temperature at a point  $(x, y)$  is  $T(x, y)$  degrees. A snail crawls so that its position after  $t$  minutes is

$$x = -2 + \frac{6}{\sqrt{1+t}}, \quad y = 7 + \sqrt{1+t}.$$

Suppose  $T_x(1, 9) = 3$  and  $T_y(1, 9) = 2$ . How fast is the temperature changing on the snail's path after 3 minutes?

(c) Show that  $\lim_{(x,y) \rightarrow (5,5)} \frac{(x-5)(y-5)}{(x-5)^2 + (y-5)^2}$  does not exist.

2. (36 pts) Consider the surface  $z = f(x, y) = (x-2)y^2$  with point  $P$  at  $(4, -1, 2)$ .

(a) Find an equation for the plane tangent to the surface at  $P$ .

(b) Find a linear approximation for  $f(x, y)$  and use it to estimate the value of  $f(4.05, -1.1)$ .

(c) Use Taylor's Formula to find an upper bound for the error in the linear approximation of  $f(x, y)$  given the values  $3.8 \leq x \leq 4.2$  and  $-1.1 \leq y \leq -0.9$ .

(d) Sketch the level curve  $z = 2$  and the vector  $\nabla f(4, -1)$  on the same axes.

3. (34 pts) Suppose the elevation of the land near Chet Chipmunk's home is given by

$$g(x, y) = \frac{x^3}{3} - \frac{y^2}{2} + 2xy + 2$$

where  $x$  and  $y$  are measured in meters.

(a) Find the critical points  $(x, y)$  where the land has local extrema or saddle points. Use the Second Derivatives Test to classify the points.

(b) A nearby trail runs along the line  $y = x$  for  $-5 \leq x \leq 3$ . At what  $x$ -coordinate does the trail have a local maximum?

(c) Chet is at  $Q(2, 0)$  when he spots a fox at  $R(0, 1)$ .

i. Find the directional derivative of  $g$  at  $Q$  in the direction of  $\overrightarrow{QR}$ .

ii. Chet decides to scramble to higher ground as quickly as possible. In which direction should he move? Express your simplified answer in terms of a unit vector.