- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/101525 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) The function $f(x,y) = x^2 + \sqrt[3]{y}$ has no critical points.

   (b) The level curves of the function $f(x,y) = \dfrac{1}{2 - 2x + x^2 + y^2}$ are circles or points.

   (c) If $f(x,y) \to 5$ as $(x,y) \to (0,0)$ along the line $y = x$ and $f(x,y) \to 5$ as $(x,y) \to (0,0)$ along the parabola $y = x^2$, then $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ must equal 5.

   (d) There exists a real number c such that $f(x,y) = \begin{cases} \dfrac{|x|}{|x|+|y|} & (x,y) \neq (0,0) \\ c & (x,y) = (0,0) \end{cases}$ is continuous throughout $\mathbb{R}^2$.

   (e) The linearization of $g(x,y) = \dfrac{2x+3}{4y+1}$ at the origin is $L(x,y) = 3 + 2x - 12y$.

2. [2350/101525 (19 pts)] Buzz and Emperor Zurg are still fighting. Zurg's energy shield is in the shape of $x^2 + 2y^2 + z^2 = 20$ and he also deployed a magnetic field having a strength of $M(x,y,z) = x + y^2 + 2z$ to try to contain Buzz.

   (a) (7 pts) Star Command wants to build a landing platform in the shape of a plane tangent to the energy shield at the point $(1,-3,1)$. Find the equation of the landing platform, writing your answer in the form $ax + by + cz = d$.

   (b) (12 pts) Buzz has been tasked to find the point(s) on the energy shield where the strength of the magnetic field is a minimum. Find this/these point(s) and the strength of the magnetic field there.

3. [2350/101525 (25 pts)] Our hummingbird has decided to visit the exam again. She is flying around in your neighbor's yard where the intensity of sweet-smelling nectar, N, is given by $N(x,y,z) = xy^2 + x^2z + yz^2$.

   (a) (7 pts) If she is at the point $P(1,-1,2)$, find the directional derivative of $N(x,y,z)$ in the direction from P to $Q(2,-1,3)$.

   (b) (4 pts) If she is at the point $R(1,1,1)$, is there a direction in which she can fly such that the rate of change of the nectar intensity is -25? If there is, find it. If not, explain why not.

   (c) (14 pts) She is now flying along the path $\mathbf{r}(t) = \langle \sin 3t, e^{2t}, \sin 2t \rangle$.

      i. (7 pts) When she is at the point $B(-1, e^{\pi}, 0)$ what is the rate of change of the nectar intensity with respect to time?

      ii. (7 pts) How fast is the nectar intensity changing with respect to distance or arc length at the point B?

4. [2350/101525 (12 pts)] Consider the function $f(x,y) = e^{1-x}\ln(1+y)$.

   (a) (4 pts) Find the domain and range of $f(x,y)$.

   (b) (8 pts) Find the second order Taylor polynomial for $f(x,y)$ centered at $(1,0)$ and use it to estimate the value of $\sqrt{e}\ln 2$.

MORE PROBLEMS BELOW/ON REVERSE

5. [2350/101525 (16 pts)] The following parts are unrelated.

(a) (8 pts) If $f(u, v) = u^v$ and $u = e^{st}$ and $v = st$, use the chain rule to find the rate of change of f with respect to s. Write your final answer in terms of s and t. No points awarded if the chain rule is not used.

(b) (8 pts) Let z be defined implicitly as a function of x and y by $z^3 + e^{xyz} = 2$. Find the rate of change of z with respect to x at the point $(0, 3, 1)$.

6. [2350/101525 (18 pts)] You need to find the maximum possible error in a first degree Taylor polynomial centered at $(1, 2)$ for a certain function $f(x, y)$. You are interested in using the linear approximation in the region $\mathcal{R}$ given by $|x - 1| \leq 1,\ |y - 2| \leq 2$. Two of your friends have already discovered that the largest values of $|f_{xy}|, |f_{yy}|$ on $\mathcal{R}$ are 0.5 and 3, respectively. This means that you only have to find the maximum and minimum values of $f_{xx}(x, y) = xy - x - y$ on $\mathcal{R}$. You could simply guess some numbers for these values but that will get you zero points towards your exam grade. For notational simplicity, let's set $f_{xx}(x, y) = g(x, y) = xy - x - y$.

(a) (4 pts) Do you know that $g(x, y)$ possesses these maximum and minimum values? Why or why not?

(b) (10 pts) Find these maximum and minimum values.

(c) (4 pts) Find the upper bound on the error in the first degree Taylor polynomial.