- This exam is worth 100 points and has 7 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”× 11” crib sheet with writing on one side.

0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: “I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/101525 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) If $\lambda = 0$ is an eigenvalue of the square matrix $\mathbf{A}$, then the system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ must be inconsistent.

   (b) The column space of an $n \times n$ nonsingular matrix is $\mathbb{R}^n$.

   (c) Given any $m \times n$ matrix $\mathbf{A}$, you can always compute $\text{Tr}\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)$ and $\text{Tr}\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)$.

   (d) Every set of n vectors in an n-dimensional vector space forms a basis.

   (e) If $\mathbf{A}$ is an $m \times n$ matrix and $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are solutions of $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$, then $\vec{\mathbf{u}} - \vec{\mathbf{v}}$ is a solution of the associated homogeneous system.

2. [2360/101525 (18 pts)] Consider the following matrices: $\mathbf{F} = \begin{bmatrix} -1 & 2 & 0 \\ 2 & -2 & 3 \end{bmatrix}$, $\mathbf{G} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} 2 & -1 \end{bmatrix}$. Compute the following if possible, making sure to fully simplify the result. If not possible to compute, write NP.

   (a) $|\mathbf{G}|$ (b) $\mathbf{H}\mathbf{F}\mathbf{F}^{\mathrm{T}}$ (c) $\text{Tr}\,\mathbf{H}$ (d) $\mathbf{G}^{\mathrm{T}}\mathbf{G}$ (e) $5\mathbf{F} - 8\mathbf{G}$ (f) $\left|\mathbf{H}^2\right|$

3. [2360/101525 (15 pts)] Use Gauss-Jordan row reduction to find the RREF and solution of the following system.

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 7 \\ x_1 + x_2 + x_3 &= 6 \\ 2x_1 \qquad + 5x_3 &= 19 \\ 3x_1 - x_2 + 2x_3 &= 11 \end{aligned}$$

4. [2360/101525 (12 pts)] Find all of the eigenvalues and eigenvectors of $\mathbf{B} = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}$. State the algebraic and geometric multiplicity of each eigenvalue.

5. [2360/101525 (15 pts)] This problem deals with the vector space $\mathbb{P}_2$.

   (a) (7 pts) Does $\mathbb{P}_2 = \text{span}\left\{3t^2 - 4, 2t, t^2 - 1\right\}$? Draw your conclusion, if possible, by computing the Wronksian.

   (b) (8 pts) Without using the Wronskian, determine if the set $\left\{2 + 2t^2, -1 + 4t + 3t^2, 3 - 5t - 2t^2\right\}$ forms a basis for $\mathbb{P}_2$.

MORE PROBLEMS BELOW/ON REVERSE

6. [2360/101525 (12 pts)] The following parts are not related.

   (a) (6 pts) Let $\mathbf{C}$ and $\mathbf{D}$ be invertible matrices. Solve for $\vec{\mathbf{x}}$ if $\mathbf{C}(\mathbf{DC})^{-1}\vec{\mathbf{x}} = \vec{\mathbf{y}}$. Simplify your answer completely.

   (b) (6 pts) For which value(s) of k, if any, will the system $\mathbf{A}^{\mathrm{T}}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ have a unique solution if $\mathbf{A} = \begin{bmatrix} 1 & 0 & k \\ 0 & k & 1 \\ k & 0 & 4 \end{bmatrix}$.

7. [2360/101525 (18 pts)] In each of the following problems, decide whether the given subset $\mathbb{W}$ of the vector space $\mathbb{V}$ is or is not a subspace of $\mathbb{V}$. Justify your answers. If not a subspace, identify at least one requirement that is not satisfied. Assume the standard definitions of vector addition and scalar multiplication in each vector space.

   (a) (6 pts) $\mathbb{V} = \mathbb{R}^2,\ \mathbb{W} = \left\{(x, y) \in \mathbb{R}^2 \,\middle|\, y = px^2, p \in \mathbb{R}\right\}$.

   (b) (6 pts) $\mathbb{V} = \mathbb{M}_{22},\ \mathbb{W}$ is the set of 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ with $a + b = c$ and $a, b, c \in \mathbb{R}$.

   (c) (6 pts) $\mathbb{V} = \mathcal{C}(-\infty, \infty),\ \mathbb{W}$ is the set of all negative functions in $\mathcal{C}(-\infty, \infty)$.