APPM 3570/STAT 3100 — Exam 1 — Fall 2025

On the front of your bluebook, write (1) **your name**, (2) **Exam 1**, (3) **APPM 3570/STAT 3100**. Correct answers with no supporting work may receive little or no credit. Books, notes and electronic devices of any kind are not allowed. Your exam should be uploaded to Gradescope in a PDF format (Recommended: **Genius Scan**, **Scannable** or **CamScanner** for iOS/Android). **Show all work, justify your answers.** __Do all problems.__ Students are required to re-write the **honor code statement** in the box below on the **first page** of their exam submission and **sign and date it**:

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work.
Signature:_____ Date:_____

1. [EXAM01] (40pts) There are 4 unrelated parts to this question. Justify your answers.

   (a) (10pts) How many ways are there to place 12 marbles of the same size in five distinct urns: (i)(5pts) if the marbles are all black? (ii)(5pts) if each marble is a different color?

   (b) (10pts) If $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B^c) = 0.4$ and $B \subseteq C$, calculate $P(A \cup B^c \cup C^c)$.

   (c) (10pts) (i)(5pts) How many different permutations of the letters in the word W A K A T A K A K A G E are there? (ii)(5pts) If you randomly select one of these permutations, what is the probability you will select a permutation that *exactly* contains the string W A K A in that order, in any part of the permutation?

   (d) (10pts) An ice cream shop has a total of seven flavors of ice cream (including chocolate and vanilla). (i)(5pts) Consider the experiment of selecting two flavors of ice cream, how many total possible outcomes are there? (ii)(5pts) Ralphie plans on getting a bowl with two *different* scoops of ice cream. What is the probability that one of the scoops she chooses will be vanilla or chocolate?

2. [EXAM01] (32pts) Customers who purchase vehicles at a certain dealership can order an engine in any of three sizes: *small*, *medium*, *large*. Of all cars sold at this dealership, 50% have the small engine and 30% have a medium-sized engine. Of cars with a medium sized engine, 20% fail emissions test within two years of purchase, while only 10% of cars fail emissions test within two years of purchase if they have the small engine. The percentage of vehicles that have the large engine *and* pass the emissions test within two years of purchase is 12%. *(Answer the questions below, justify your answers.)*

   (a) (8pts) Given that a vehicle from the dealership has a large engine, what is the probability it will pass the emissions tests within two years of purchase?

   (b) (8pts) What is the probability that a randomly chosen car from the dealership will fail an emissions test within two years of purchase?

   (c) (8pts) Of the next 100 cars sold at the dealership, what is the probability that 60 of them will fail an emissions test within two years of purchase? *State any assumptions you have made.*

   (d) (8pts) A vehicle that *passed* the emissions test is chosen at random, what is the probability that it has a small engine?

3. [EXAM01] (28pts) An experiment consists of flipping a coin until the first head appears or until a total of 5 flips is made. Assume the probability of getting a head on each flip is $1 - q$ and the probability of a tail on each flip is $q \in [0, 1]$.

   (a) (7pts) What is the sample space for this experiment? **Give your answer in set notation.**

   (b) (7pts) Let Y be the random variable that counts the number of flips. Find the *probability mass function* (pmf) of Y. (Your pmf should be defined for all real numbers.) *State any assumptions you are making*.

   (c) (7pts) Verify that your answer from part (b) is indeed a probability mass function. Show all work.

   (d) (7pts) For what value of $q \in [0, 1]$ will the likelihood of event $\{Y = 4\}$ occurring be maximized? Justify your answer.