Write your name and your professor's name or your section number below. You are *not* allowed to use textbooks, notes, or a calculator. To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise.

Name:

Instructor and Section:

1. (24 pts) If the statement is **always true**, write "TRUE"; if it is possible for the statement to be false then mark "FALSE." You must give a **justification** for your answer. That is, if the answer is true, provide a brief proof. If the answer is false, provide a counterexample.

   (a) The vectors $\vec{v}_1 = (1, 0, 1)^T$, $\vec{v}_2 = (0, 1, 1)^T$, and $\vec{v}_3 = (0, 0, 1)^T$ form a basis for $\mathbb{R}^3$.

   (b) If A and B are nonsingular $n \times n$ matrices, then $(A + B)^{-1} = A^{-1} + B^{-1}$

   (c) If A and B are symmetric $n \times n$ matrices then AB is symmetric.

   (d) The set of vectors $(x, y, z)^T$ with $x, y, z \in \mathbb{R}$ such that $x + y + z = 1$ is a vector subspace.

2. (21 pts) Let A be the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 & 1 \\ 2 & 1 & 1 & 1 \\ 3 & 2 & c & 1 \end{pmatrix}$$

for c an arbitrary real parameter.

   (a) (5 pts) Reduce A to a matrix U in Row-Echelon form.

   (b) (5 pts) What is rank(A)? How does it depend on c?

   (c) (5 pts) Solve the homogeneous problem $A\vec{x} = \vec{0}$. Does $A\vec{x} = \vec{0}$ always have a solution? Explain why or why not.

   (d) (6 pts) Solve the inhomogeneous problem $A\vec{x} = (1, 1, 1)^T$. Does this have a solution for all values of c? Explain why or why not.

3. (15 pts)

   (a) Construct a basis for the vector space of real skew-symmetric 3×3 matrices, $A^T = -A$. What is the dimension of this space?

   (b) Do the same (i.e., construct a basis) for the vector space of real symmetric 3×3 matrices, $A^T = A$. What is its dimension?

   (c) Any real 3×3 matrix can be written as a sum of symmetric and skew-symmetric matrices,

   $$A = \tfrac{1}{2}\left(A + A^T\right) + \tfrac{1}{2}\left(A - A^T\right).$$

   What is the dimension of the vector space of all real 3×3 matrices?

4. (20 pts) For the matrix

$$A = \begin{pmatrix} 0 & 2 & -2 & 2 \\ -2 & 1 & -1 & -1 \\ -1 & 1 & -1 & 0 \end{pmatrix}$$

find bases for the

(a) (7 pts) image of A and

(b) (7 pts) kernel of A.

(c) (6 pts) Is $(6, -1, 1)^T$ in the image of A?

5. (20 pts) Let

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

(a) (7 pts) Find the permutation matrix P such that PA is symmetric.

(b) (7 pts) Find matrices L and D such that $PA = LDL^T$.

(c) (6 pts) What is the determinant of A?