1. (18 pts) The following questions are not related.

   (a) (6 points) Consider the rational function

   $$g(t) = \frac{t^3 - 2t - 4}{t^2(t-2)^3(t^2+t+1)^2}$$

   Write out the form for the partial fraction decomposition, but do not solve for the coefficients.

   (b) (6 points) Suppose that you have used the trig substitution $x = a\sin\theta$ (a is a positive constant) to integrate $y = f(x)$ and obtained

   $$\int f(x)\,dx = \frac{a^2\theta}{2} + \frac{a^2}{2}\sin(\theta)\cos(\theta) + C$$

   What is the final solution to the integral in terms of x?

   (c) (6 points) **True or false:** If f is a continuous, decreasing function on $[1,\infty)$ with $\lim_{x\to\infty} f(x) = 0$, then $\int_1^\infty f(x)\,dx$ is convergent. *You must justify your answer: if the statement is true, explain why. If it is false, you need to find an example that shows it is false.*

2. (34 pts) Evaluate the following integrals, and simplify your answers.

   (a) (10 pts) $\displaystyle\int \sin^3(\theta)\cos^2(\theta)\,d\theta$

   (b) (12 pts) $\displaystyle\int_7^{7\sqrt{2}} \frac{\sqrt{t^2-49}}{t}\,dt$

   (c) (12 pts) $\displaystyle\int_1^4 x^{1/2}\ln(x)\,dx$

3. (24 pts) Determine whether the following integrals are convergent or divergent. Explain your reasoning fully for each integral. If an integral can be evaluated, please do so. (**If you use the Comparison Test, state this and evaluate the integral that you are using for comparison.**)

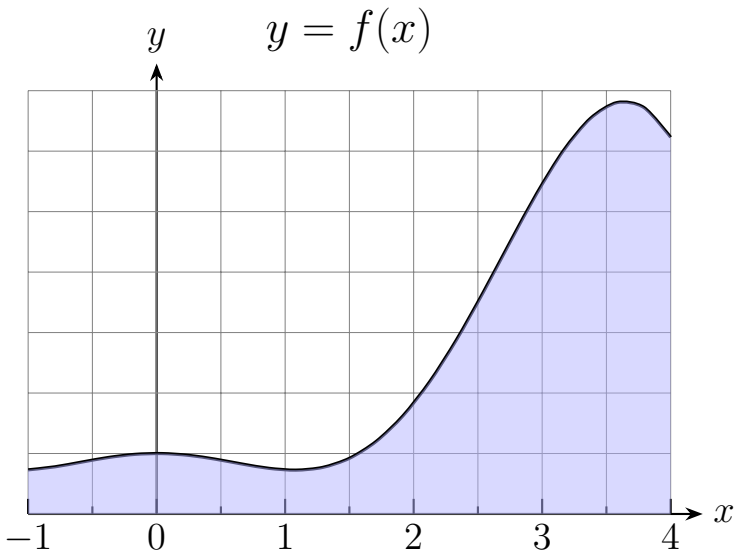
   (a) $\displaystyle\int_2^\infty \frac{1}{(u-1)(u+3)}du$

   (b) $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$

   (c) $\displaystyle\int_3^\infty \frac{x^2}{x^{5/2}-2}\,dx$

4. (24 pts) All parts of this problem refer to the same function f, whose graph is below. The left, right, trapezoidal, and midpoint rule approximations were used to estimate $\displaystyle\int_{-1}^4 f(x)\,dx$.

   For $n = 2$ subintervals, the estimates were 8.25, 22.00, 27.32, and 35.75.



(a) Which rule produced which estimate? (Write down L_2, R_2, T_2, and M_2 and match each to its estimate.)

(b) For each of the four estimates, state whether it is an underestimate or an overestimate of the true value of $\int_{-1}^{4} f(x)\,dx$.

(c) Suppose the number of subintervals in the trapezoidal approximation is increased to $n = 50$, and suppose $f''(x) = -2\cos x + x^2\cos x + 4x\sin x$. Estimate the error in T_{50}. You do not need to simplify the values in your numerator and denominator.

(d) Now suppose you want to estimate the value of the integral with error less than 10^{-6} using the trapezoidal rule. What value of n should you use? (Use $f''(x)$ from part (c). You should solve for n but you do not need to simplify your answer further.)