- This exam is worth 100 points and has 7 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”× 11” crib sheet with writing on one side.

0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: “I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/091725 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) If $\mathbf{r}(t) = \left\langle e^{-t}\cos t, e^{-2t}\sin 2t, \dfrac{2}{t}+1 \right\rangle$, then $\lim_{t\to\infty} \mathbf{r}(t) = \mathbf{k}$.

   (b) The vector $3\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$ is parallel to the plane $6x - 2y + 4z = 1$.

   (c) For any vectors $\mathbf{u}, \mathbf{v}$, $|\mathbf{u}\cdot\mathbf{v}| \le \|\mathbf{u}\|\|\mathbf{v}\|$.

   (d) If $\mathbf{v}$ is a nonzero vector, then $\mathbf{r}(t) = \langle 1, 2, 3\rangle + t\dfrac{\mathbf{v}}{\|\mathbf{v}\|}$ is an arc length parameterization.

   (e) If $\mathbf{T}(t)$ is the unit tangent vector of a smooth curve, then the curvature is $\kappa = \|\mathrm{d}\mathbf{T}/\mathrm{d}t\|$.

2. [2350/091725 (14 pts)] Buzz Lightyear is positioned at $(0, 1, 0)$ and is attempting to return to his Star Command Base. Unfortunately, the Emperor Zurg has placed an energy shield in the shape of $x^2 + 2y^2 + z^2 = 10$ in the region surrounding Buzz to try and trap him. However, he received word from Star Command that he can safely escape the energy shield at two intersection points if he travels along the path $\mathbf{r}(t) = t\,\mathbf{i} + (1+t)\,\mathbf{j} - t\,\mathbf{k}$ where t is a real number. Find the closest such intersection point to Buzz’s present position.

3. [2350/091725 (19 pts)] A hummingbird is flying along the path $\mathbf{r}(t) = \left\langle \dfrac{\sqrt{2}}{2}t^2, t+1, -\dfrac{1}{3}t^3 \right\rangle$, where $t \ge 0$.

   (a) (3 pts) Find the hummingbird’s velocity vector.

   (b) (3 pts) Verify that the hummingbird’s speed is given by $t^2 + 1$ and then find how far the hummingbird flies during the interval $0 \le t \le 3$.

   (c) (3 pts) Find $\mathbf{T}(t)$ for the hummingbird.

   (d) (3 pts) Is the hummingbird’s acceleration vector always parallel to the xz-plane? Justify your answer.

   (e) (7 pts) Given that $\mathbf{N}(t) = \dfrac{1}{t^2+1}\left\langle 1 - t^2, -\sqrt{2}t, -\sqrt{2}t \right\rangle$, which you need not verify, find the equation of the hummingbird’s osculating plane when $t = \sqrt{2}$, writing your answer in the form $ax + by + cz = d$.

4. [2350/091725 (14 pts)] Find the equation of the plane (write your answer in the form $ax + by + cz = d$) containing the intersecting lines

$$L_1 : x = 2 - s, y = s, z = 2 \quad \text{and} \quad L_2 : x - 1 = 1 - y = \frac{z}{2}$$

MORE PROBLEMS BELOW/ON REVERSE

5. [2350/091725 (16 pts)] You are trapped in a box with a hinged lid on top. Because you are in such a tight space, you can only push up on the lid at a 45 degree angle using a force, $\mathbf{F}$, of magnitude 2.

   (a) (4 pts) If the magnitude of the torque, $\boldsymbol{\tau}$, required to rotate the lid around the hinge, opening the box, is $\sqrt{6}$, how far from the hinge should you apply the force?

   (b) (4 pts) If the same force applied at the same angle moves an object the same distance as that found in part (a), would the work be the same as the magnitude of the torque? Justify your answer.

   (c) (4 pts) Find the area of the parallelogram formed by the force vector and the vector depicting where you applied the force in part (a).

   (d) (4 pts) Is the work found in part (b) equal to $\text{proj}_{\mathbf{D}}\mathbf{F}$ where $\mathbf{D}$ is the vector giving the displacement of the object? Justify your answer.

6. [2350/091725 (15 pts)] A particle is traveling along a path, $\mathbf{r}(t)$, experiencing an acceleration of $\mathbf{a}(t) = 2t\,\mathbf{i} + e^t\,\mathbf{j}$. Assume that $t \geq 0$ and $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$.

   (a) Find the time(s), if any, when the speed of the particle is not changing.

   (b) Is the direction of the particle always changing? Justify your answer.

7. [2350/091725 (12 pts)] Consider the equation $ax^2 + by^2 + cz^2 + 2z = 2$. For each part, pick the correct surface from the list below that is described when the constants take on the given values. Write your answers in a single column separate from any work you do to arrive at your answers. No partial credit will be awarded.

   cylinder   hyperboloid of one sheet   paraboloid   ellipsoid   plane   cone   hyperboloid of two sheets

   (a) $a > 0, b > 0, c = -1/2$

   (b) $a \neq 0, b = c = 0$

   (c) $a \neq 0, b \neq 0, c = 0$

   (d) $a = b = c = 0$

   (e) $a = 1, b = c = -1$

   (f) $a = c = 1, b = -1$