1. The following are unrelated: (18 pts)

(a) Consider the comma-separated list of numbers $\left\{\frac{1}{6}, \sqrt{4}, -\frac{3}{2}, 0, \pi, \sqrt{2}, \frac{8}{40}\right\}$ and answer the following:

i. Write down all rational numbers given in the list. **Include all expressions that simplify to a rational number.**

Solution: $\boxed{\left\{\frac{1}{6},\ \sqrt{4},\ -\frac{3}{2},\ 0,\ \frac{8}{40}\right\}}$ since $\sqrt{4} = 2$.

ii. Write down the numbers given in the comma-separated list (include all numbers, not just rational numbers) from smallest to largest.

Solution: $\boxed{-\frac{3}{2},\ 0,\ \frac{1}{6},\ \frac{8}{40},\ \sqrt{2}, \sqrt{4},\ \pi}$ since

- $-\frac{3}{2}$ is the only negative number,
- 0 is smaller than positive numbers and bigger than negative numbers,
- $\frac{1}{6} < \frac{8}{40}$ as $\frac{8}{40} = \frac{1}{5}$,
- $\sqrt{2} > 1$, therefore $\sqrt{2} > \frac{8}{40}$,
- $\sqrt{2} < \sqrt{4} = \sqrt{2^2}$,
- $\sqrt{4} < \pi$ as $\sqrt{4} = 2$ and $\pi \approx 3.14159$.

(b) Given $x > 0$, $y < 0$, and $z < 0$, determine whether each expression is positive, negative, or zero.

i. $2x^3y^4z$

Solution: $2x^3y^4z$ is negative as $x^3 > 0$, $y^4 > 0$, and $z < 0$. $\boxed{Negative}$

ii. $-y^4z^5$

Solution: $-y^4z^5$ is positive as $y^4 > 0$ and $z^5 < 0$. Therefore, $y^4z^5 < 0$ and $-y^4z^5 > 0$. $\boxed{Positive}$

iii. $y^2 + 1$

Solution: $y^2 + 1$ is positive as $y^2 > 0$. $\boxed{Positive}$

(c) Rewrite the expression without using absolute value:

i. $|5 - x|$ if $x < 5$

Solution: When $x < 5$, $5 - x > 0$, therefore $|5 - x| = \boxed{5 - x}$.

ii. $|2 - \pi|$

Solution: As $\pi \approx 3.14159$, $2 - \pi < 0$, therefore $|2 - x| = \boxed{-(2 - \pi)}$ or $\boxed{\pi - 2}$.

(d) Subtract/add as indicated: $\dfrac{2}{15} - \dfrac{1}{9} + 3^{-1}$

Solution:

$$\begin{aligned}
\frac{2}{15} - \frac{1}{9} + 3^{-1} &= \frac{2}{15} - \frac{1}{9} + \frac{1}{3} && (1)\\
&= \frac{2}{15}\cdot\frac{3}{3} - \frac{1}{9}\cdot\frac{5}{5} + \frac{1}{3}\cdot\frac{15}{15} && (2)\\
&= \frac{6}{45} - \frac{5}{45} + \frac{15}{45} && (3)\\
&= \frac{6-5+15}{45} && (4)\\
&= \boxed{\frac{16}{45}} && (5)
\end{aligned}$$

2. The following are unrelated: (18 pts)

(a) Simplify: $(2x+1)^2 - 2\left(\dfrac{3}{2}x^2 - 5x\right)$

Solution:

$$\begin{aligned}
(2x+1)^2 - 2\left(\frac{3}{2}x^2 - 5x\right) &= (2x+1)\cdot(2x+1) - (2\cdot\frac{3}{2}x^2 - 2\cdot 5x) && (6)\\
&= (2x)^2 + 2x + 2x + 1^2 - (3x^2 - 10x) && (7)\\
&= 4x^2 + 4x + 1 - 3x^2 + 10x && (8)\\
&= \boxed{x^2 + 14x + 1} && (9)
\end{aligned}$$

(b) Simplify: $\dfrac{8x^3y}{4x^{-3}y^{-9}} + \left(3x^3y^5\right)^2$

Solution:

$$\begin{aligned}
\frac{8x^3y}{4x^{-3}y^{-9}} + \left(3x^3y^5\right)^2 &= \frac{2x^3y}{x^{-3}y^{-9}} + 3^2x^{3\cdot 2}y^{5\cdot 2} && (10)\\
&= 2x^{3+3}y^{1+9} + 9x^6y^{10} && (11)\\
&= 2x^6y^{10} + 9x^6y^{10} && (12)\\
&= \boxed{11x^6y^{10}} && (13)
\end{aligned}$$

(c) Find the missing power (that value of the exponent in the box that would make the equality true) in the calculation: $x^{2/7}\cdot x^{\square} = x$

Solution:

$$\begin{aligned}
x^{2/7}\cdot x^{\square} &= x && (14)\\
\frac{x^{2/7}\cdot x^{\square}}{x^{2/7}} &= \frac{x}{x^{2/7}} && (15)\\
x^{\square} &= x^{1-\frac{2}{7}} && (16)\\
x^{\square} &= x^{1\cdot\frac{7}{7}-\frac{2}{7}} && (17)\\
x^{\square} &= x^{\frac{7-2}{7}} && (18)\\
x^{\square} &= x^{\frac{5}{7}} && (19)
\end{aligned}$$

The missing power is $\boxed{\dfrac{5}{7}}$.

(d) Simplify each expression:

i. $\sqrt{72} - \sqrt{32}$

Solution:

$$\begin{aligned}
\sqrt{72} - \sqrt{32} &= \sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} - \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} && (20)\\
&= \sqrt{2^2 \cdot 2 \cdot 3^2} - \sqrt{2^2 \cdot 2^2 \cdot 2} && (21)\\
&= 2 \cdot 3\sqrt{2} - 2 \cdot 2\sqrt{2} && (22)\\
&= 6\sqrt{2} - 4\sqrt{2} && (23)\\
&= \boxed{2\sqrt{2}} && (24)
\end{aligned}$$

ii. $\sqrt{4x^2 + 16}$

Solution:

$$\begin{aligned}
\sqrt{4x^2 + 16} &= \sqrt{4(x^2 + 4)} && (25)\\
&= \sqrt{4}\sqrt{(x^2 + 4)} && (26)\\
&= \sqrt{2^2}\sqrt{(x^2 + 4)} && (27)\\
&= \boxed{2\sqrt{x^2 + 4}} && (28)
\end{aligned}$$

(e) Multiply: $x^{1/3}\left(x^{2/3} + \frac{1}{\sqrt[3]{x}}\right)$

Solution:

$$\begin{aligned}
x^{1/3}\left(x^{2/3} + \frac{1}{\sqrt[3]{x}}\right) &= x^{1/3} \cdot x^{2/3} + x^{1/3} \cdot \frac{1}{\sqrt[3]{x}} && (29)\\
&= x^{1/3+2/3} + x^{1/3} \cdot \frac{1}{x^{1/3}} && (30)\\
&= x^{\frac{1+2}{3}} + 1 && (31)\\
&= x^{3/3} + 1 && (32)\\
&= \boxed{x + 1} && (33)
\end{aligned}$$

3. The following are unrelated: (20 pts)

(a) Factor completely (If not factorable write NF): $27x^3 - 1$

Solution: We use our difference of cubes formula provided on the exam with $a^3 = (3x)^3$ so $a = 3x$ and $b^3 = 1^3$ so $b = 1$:

$$\begin{aligned}
27x^3 - 1 &= (3x - 1)((3x)^2 + (3x)(1) + (1)^2) && (34)\\
&= \boxed{(3x - 1)(9x^2 + 3x + 1)} && (35)
\end{aligned}$$

NOTE: $9x^2 + 3x + 1$ is not factorable so we are done.

(b) Simplify the compound fraction: $\dfrac{\frac{1}{2x^2} - \frac{4}{x}}{\frac{1}{3x^2} - 3}$

Solution: One approach to solving this is to clear the fractions by multiplying both the numerator and denominator by the common denominator of all the three fractions, then simplify:

$$\frac{\frac{1}{2x^2} - \frac{4}{x}}{\frac{1}{3x^2} - 3} = \left(\frac{\frac{1}{2x^2} - \frac{4}{x}}{\frac{1}{3x^2} - 3}\right)\left(\frac{6x^2}{6x^2}\right) \tag{36}$$

$$= \left(\frac{\frac{6x^2}{2x^2} - \frac{4(6x^2)}{x}}{\frac{6x^2}{3x^2} - 3(6x^2)}\right) \tag{37}$$

$$= \boxed{\frac{3 - 24x}{2 - 18x^2}} \tag{38}$$

$$\text{or } = \boxed{\frac{3(1 - 8x)}{2(1 - 9x^2)}} \tag{39}$$

$$\text{or } = \boxed{\frac{3(1 - 8x)}{2(1 - 3x)(1 + 3x)}} \tag{40}$$

Since nothing cancels when we factor, any of the three answers are correct.

(c) Factor completely (If not factorable write NF): $x^3 - 4x^2 + 2x - 8$

$$x^3 - 4x^2 + 2x - 8 = x^2(x - 4) + 2(x - 4) \tag{41}$$

$$= \boxed{(x^2 + 2)(x - 4)} \tag{42}$$

(d) Let d be a constant real number. Find the value of d that makes the factoring of the polynomial true: $3x^2 + dx - 8 = (3x + 1)(x - 8)$

Solution: We distribute on the right hand side of our equation and solve for d:

$$3x^2 + dx - 8 = (3x + 1)(x - 8) \tag{43}$$

$$3x^2 + dx - 8 = 3x^2 - 24x + x - 8 \tag{44}$$

$$3x^2 + dx - 8 = 3x^2 - 23x - 8 \tag{45}$$

$$dx - 8 = -23x - 8 \tag{46}$$

$$dx = -23x \tag{47}$$

$$d = \boxed{-23} \tag{48}$$

4. Simplify: $\dfrac{(x - 2)(-3)4x^2 + (2x)^2 2x}{x}$ (5 pts)

Solution:

$$\frac{(x - 2)(-3)4x^2 + (2x)^2 2x}{x} = \frac{x\left((x - 2)(-3)4x + (2x)^2 2\right)}{x} \tag{49}$$

$$= (x - 2)(-3)4x + 2(2x)^2 \tag{50}$$

$$= (x - 2)(-12x) + 2(4x^2) \tag{51}$$

$$= -12x^2 + 24x + 8x^2 \tag{52}$$

$$= \boxed{-4x^2 + 24x} \tag{53}$$

5. The following are unrelated: (10 pts)

   (a) Perform the subtraction: $\dfrac{1}{x^2+5x} - \dfrac{2}{x+5}$

   Solution: We begin by finding the common denominator by factoring:

$$\frac{1}{x^2+5x} - \frac{2}{x+5} = \frac{1}{x(x+5)} - \frac{2}{x+5} \tag{54}$$

$$= \frac{1}{x(x+5)} - \frac{2x}{x(x+5)} \tag{55}$$

$$= \boxed{\frac{1-2x}{x(x+5)}} \tag{56}$$

   (b) Perform the multiplication: $\dfrac{2x^4+8x^2}{4\left(x^2-6x+9\right)} \cdot \dfrac{x-3}{x^2+4}$

   Solution: We factor and cancel:

$$\frac{2x^4+8x^2}{4\left(x^2-6x+9\right)} \cdot \frac{x-3}{x^2+4} = \frac{2x^2(x^2+4)}{4(x-3)(x-3)} \cdot \frac{x-3}{x^2+4} \tag{57}$$

$$= \boxed{\frac{x^2}{2(x-3)}} \tag{58}$$

6. Is $x = 9$ a solution of the equation: $\dfrac{\sqrt{x}}{x-10} + 2x = 14$? As usual, be sure to show work to justify your answer for credit. (4 pts)

   Solution:
   Plugging $x = 9$ to the left hand side (LHS) of the equation, we obtain

$$\frac{\sqrt{9}}{9-10} + 18 = \frac{3}{-1} + 18 \tag{59}$$

$$= 15 \tag{60}$$

   Since 15 does not match the right hand side (RHS) value of 14, we conclude that $\boxed{x = 9 \text{ is NOT a solution of the given equation.}}$

7. Solve each of the following equations: (15 pts)

   (a) $12 + 8x = -x^2$

   Solution:

$$12 + 8x = -x^2 \tag{61}$$

$$x^2 + 8x + 12 = 0 \tag{62}$$

$$(x+6)(x+2) = 0 \tag{63}$$

   Thus, using the multiplicative property of zero, we conclude that $\boxed{x = -6}$ and $\boxed{x = -2}$

(b) $\frac{1}{4}x - 2 = \frac{5}{6} - 2x$

Solution:

$$\frac{1}{4}x - 2 = \frac{5}{6} - 2x \quad (64)$$

$$12\left(\frac{1}{4}x - 2\right) = 12\left(\frac{5}{6} - 2x\right) \quad (65)$$

$$3x - 24 = 10 - 24x \quad (66)$$

$$27x = 34 \quad (67)$$

$$x = \boxed{\frac{34}{27}} \quad (68)$$

(c) $\left(y^2 + 9\right)\left(2y^2 - 4\right) = 0$

Solution:
Using the multiplicative property of zero, the possible solutions come from:

i. $y^2 + 9 = 0$

$$y^2 + 9 = 0 \quad (69)$$

$$y^2 = -9 \quad (70)$$

$$y = \pm\sqrt{-9} \quad (71)$$

but $\sqrt{-9}$ and $-\sqrt{-9}$ are not real numbers (and so $y^2 + 9 = 0$ has no solutions in the real numbers).

ii.

$$2y^2 - 4 = 0 \quad (72)$$

$$2y^2 = 4 \quad (73)$$

$$y^2 = 2 \quad (74)$$

$$y = \pm\sqrt{2} \quad (75)$$

So the solutions to $\left(y^2 + 9\right)\left(2y^2 - 4\right) = 0$ are $\boxed{y = \pm\sqrt{2}}$

8. Solve each of the following Physics equations for the specified variable: (10 pts)

(a) Solve for t: $2s - vt = 2at$
Solution:

$$2s - vt = 2at \quad (76)$$

$$2s = vt + 2at \quad (77)$$

$$2s = t(v + 2a) \quad (78)$$

$$t = \boxed{\frac{2s}{v + 2a}} \quad (79)$$

(b) Solve for v: $T = \frac{1}{2}mv^2$
Solution:

$$T = \frac{1}{2}mv^2 \quad (80)$$

$$2T = mv^2 \quad (81)$$

$$\frac{2T}{m} = v^2 \quad (82)$$

$$v = \boxed{\pm\sqrt{\frac{2T}{m}}} \quad (83)$$