APPM 2360 - Introduction to Differential Equations with Linear Algebra

COURSE OBJECTIVES: Provide an introduction to ordinary differential equations and linear algebra. The main objectives are to:

- Understand various qualitative and quantitative methods of solving differential equations
- Understand basic concepts of linear algebra and utilize these to solve differential equations
- Improve problem solving and critical thinking skills

TEXTBOOK:

- *Differential Equations and Linear Algebra*, 2nd Edition by Farlow, Hall, McDill and West
- Digital resources are also available through WebAssign

PREREQUISITES:

- Any ONE of the following courses (minimum grade C-): APPM 1360 or MATH 2300

EQUIVALENT COURSES: Duplicate Degree Credit Not Granted:

- both MATH 2130 and MATH 3430

SCHEDULE AND TOPICS COVERED

Day	Section	Topics
1	1.2	Solutions and Direction Fields: Qualitative Analysis [a]
2	1.2 / 1.3	Solutions and Direction Fields: Qualitative Analysis [b] / Separation of Variables: Quantitative Analysis [a]
3	1.3 / 1.4	Separation of Variables: Quantitative Analysis [b] / Approximation Methods: Numerical Analysis
4	1.5 / 2.1	Picard's Theorem: Theoretical Analysis / Linear Equations: The Nature of Their Solutions [a]
5	2.1 / 2.2	Linear Equations: The Nature of Their Solutions [b] / Solving the First Order Linear Differential Equation [a]
6	2.2 / 2.3	Solving the First Order Linear Differential Equation [b] / Growth and Decay Phenomena [a]
7	2.3 / 2.4	Growth and Decay Phenomena [b] / Linear Models: Mixing and Cooling [a]
8	2.4 / 2.5	Linear Models: Mixing and Cooling [b] / Nonlinear Models: Logistic Equation [a]
9	2.5 / 2.6	Nonlinear Models: Logistic Equation [b] / Systems of Differential Equations: A First Look [a]
10	2.6	Systems of Differential Equations: A First Look [b]
11	Review	Exam Review (Sections 1.2-2.6)
12	3.1	Matrices: Sums and Products [a]
13	3.1 / 3.2	Matrices: Sums and Products [b] / Systems of Linear Equations [a]
14	3.2	Systems of Linear Equations [b]
15	3.2 / 3.3	Systems of Linear Equations [c] / The Inverse of a Matrix [a]
16	3.3 / 3.4	The Inverse of a Matrix [b] / Determinants and Cramer's Rule [a]
17	3.4 / 3.5	Determinants and Cramer's Rule [b] / Vector Spaces and Subspaces [a]
18	3.5 / 3.6	Vector Spaces and Subspaces [b] / Basis and Dimension [a]
19	3.6	Basis and Dimension [b]
20	3.6	Basis and Dimension [c]
21	3.6 / 5.3	Basis and Dimension [d] / Eigenvalues and Eigenvectors [a]
22	5.3	Eigenvalues and Eigenvectors [b]
23	Review	Exam Review (Sections 3.1-3.6, 5.3)
24	4.1	The Harmonic Oscillator
25	4.2	Real Characteristic Roots
26	4.3	Complex Characteristic Roots
27	4.4	Undetermined Coefficients [a]
28	4.4 / 4.5	Undetermined Coefficients [b] / Variation of Parameters [a]
29	4.5 / 4.6	Variation of Parameters [b] / Forced Oscillations [a]
30	4.6 / 4.7	Forced Oscillations [b] / Conservation and Conversion [a]
31	4.7 / 8.1	Conservation and Conversion [b] / The Laplace Transform and Its Inverse [a]
32	8.1 / 8.2	The Laplace Transform and Its Inverse [b] / Solving DEs and IVPs with Laplace Transforms [a]
33	8.2	Solving DEs and IVPs with Laplace Transforms [b]

34	8.3	The Step Function and the Delta Function [a]
35	Review	Exam Review (Sections 4.1-4.7, 8.1-8.2)
36	8.3	The Step Function and the Delta Function [b]
37	6.1 / 6.2	Theory of Linear DE Systems / Linear Systems with Real Eigenvalues [a]
38	6.2	Linear Systems with Real Eigenvalues [b]
39	6.2 / 6.3	Linear Systems with Real Eigenvalues [c] / Linear Systems with Nonreal Eigenvalues [a]
40	6.3 / 6.4	Linear Systems with Nonreal Eigenvalues [b] / Stability and Linear Classification [a]
41	6.4	Stability and Linear Classification [b]
42	Review	Final Exam Review (comprehensive)

Learning Objectives by Section

Section	Topics	Learning Objectives – After completing this section, students should be able to do the following:
1.2	Solutions and Direction Fields: Qualitative Analysis	• know what an ordinary differential equation (ODE) is and how to determine its order • know what a solution to a differential equation is and how to verify that a function is a solution of an ODE on an interval • know what an initial value problem (IVP) is • know the difference between a general and particular solution of an ODE/IVP • create direction fields and use them to analyze behavior of solutions of differential equations • find equilibrium solutions • classify the stability of equilibrium solutions • determine the long term behavior of ODE solutions based on direction fields alone • use isoclines to generate direction fields
1.3	Separation of Variables: Quantitative Analysis	• identify separable differential equations • perform separation of variables to find explicit and implicit solutions to differential equations • use variable transformations/substitutions to make nonseparable equations separable
1.4	Approximation Methods: Numerical Analysis	• understand and implement Euler's Method for approximating solutions to differential equations • qualitatively understand error types (roundoff/discretization) and error propagation in Euler's Method • other methods discussed such as Runge-Kutta
1.5	Picard's Theorem: Theoretical Analysis	• understand existence and uniqueness of solutions • apply Picard's theorem to determine if an IVP has a unique solution • understand when Picard's theorem is not applicable

2.1	Linear Equations: The Nature of Their Solutions	• define linearity for differential and algebraic equations • recognize linear and nonlinear algebraic and differential equations • understand linear operators and their properties • distinguish between homogeneous and nonhomogeneous linear equations • understand the Superposition Principle • understand the Nonhomogeneous Principle • know how to build solutions to nonhomogeneous linear equations
2.2	Solving the First Order Linear Differential Equation	• use Euler-Lagrange Two Stage Method (variation of parameters) to solve first order linear nonhomogeneous differential equations • find and use an integrating factor to solve first order linear nonhomogeneous differential equations • analyze solutions in terms of transient and steady state behavior
2.3	Growth and Decay Phenomena	• apply linear differential equations to exponential decay and growth problems • solve applied problems concerning compound interest • apply similar equations to other growth and decay problems
2.4	Linear Models: Mixing and Cooling	• apply physical principles to model mixing problems with differential equations • solve mixing problem differential equations • set up and solve applied problems using Newton's law of cooling
2.5	Nonlinear Models: Logistic Equation	• perform qualitative analysis of nonlinear differential equations • identify equilibrium solutions and determine their stability • identify autonomous and non-autonomous differential equations • plot phase lines for autonomous equations • solve and interpret initial value problems for the logistic, threshold and similar equations
2.6	Systems of Differential Equations: A First Look	• define what a system of differential equations is • understand what a solution to a system of differential equations is • know how to create phase portraits for autonomous systems of two differential equations • define and find equilibrium solutions and their stability for systems of differential equations • define and sketch nullclines • understand uniqueness of solutions and its relation to trajectories • analyze various models of systems of differential equations (*e.g.*, predator-prey and competition models)

3.1	Matrices: Sums and Products	• understand basic properties and terminology of matrices including – order/size of a matrix – rows, columns and their relation to subscript notation of elements – zero matrix – identity matrix – diagonal and triangular matrices – matrices with functions as entries • recognize column and row vectors as special matrices • perform matrix operations and understand the properties of these operations – matrix transpose – trace of a matrix – addition of matrices and scalar multiplication – multiplication of matrices – differentiation of matrices
3.2	Systems of Linear Equations	• recognize systems of linear algebraic equations and their solutions • recognize underdetermined, overdetermined and square systems and the solution possibilities of each • recognize consistent and inconsistent systems • convert systems of linear algebraic equations into matrix/vector equations • know and apply the elementary row operations • recognize reduced row echelon form (RREF) of a matrix, leading/basic and free variables, pivot columns • perform Gauss-Jordan algorithm on augmented matrices to find solutions to systems of linear algebraic equations • determine when solutions to a linear system of algebraic equations are unique • apply Superposition and Nonhomogeneous Principles to solutions of linear algebraic equations • identify the rank of a matrix
3.3	The Inverse of a Matrix	• definition of matrix inverse • compute the inverse of a matrix • determine when a matrix inverse exists • know properties of an invertible matrix • use the inverse of a matrix to solve a system of linear algebraic equations

3.4	Determinants and Cramer's Rule	• compute determinants of 2×2 matrices • know the definition of minors and cofactors • compute determinants of $n \times n$ matrices using cofactor expansion • understand the effects of elementary row operations on determinants • know the properties of determinants • use Cramer's rule to determine unique solutions to linear systems via determinants • relate determinants with invertibility of a matrix and the number of solutions to linear systems
3.5	Vector Spaces and Subspaces	• understand the concept of vector spaces and their defining properties/axioms • identify important vector spaces • understand and apply the vector subspace theorem • identify when a set is not a vector space or subspace
3.6	Basis and Dimension	• know the definition of linear combination and span • define the column space of a matrix and its relationship to solutions of linear systems • determine when a set of vectors is linearly independent • use the Wronskian to determine if a set of functions is linearly independent • define basis for a vector space and know how to determine if a set of vectors forms a basis • define dimension of a vector space and compute it
5.3	Eigenvalues and Eigenvectors	• know the definition of an eigenvalue and eigenvector of a matrix • find the characteristic equation of a matrix • compute eigenvalues and eigenvectors and find bases for eigenspaces • understand algebraic and geometric multiplicity of eigenvalues • understand basic properties of eigenvalues
4.1	The Harmonic Oscillator	• understand the properties of the DE and initial conditions describing an harmonic oscillator • motivate and understand the importance of the harmonic oscillator equation in specific relation to physics • solve the differential equation describing an undamped harmonic oscillator • write solutions to the undamped oscillator equation in multiple forms • know the meanings of amplitude, phase angle, circular/angular frequency, natural frequency, period, phase shift • understand the impact of various coefficient choices in the governing differential equation • convert the harmonic oscillator equation to a system of first order equations • understand phase portraits for harmonic oscillators (phase plane, vector field, trajectory)

4.2	Real Characteristic Roots	• solve constant coefficient second order linear DE with real roots via the characteristic equation • deal with repeated real roots to the characteristic equation • understand overdamping and critical damping of harmonic oscillators • know the conditions that guarantee unique solutions to n^{th} order linear homogeneous DE • determine the basis and dimension of the solution space to second order constant coefficient linear DE with real roots of the characteristic equation • use the Wronskian to determine if solutions of linear homogeneous DE are linearly independent • understand and apply the solution space theorem
4.3	Complex Characteristic Roots	• determine general form of solutions to second order linear constant coefficient DEs when the characteristic polynomial has complex roots • determine the basis and dimension of the solution space to second order constant coefficient linear DE with complex roots of the characteristic equation • understand underdamping of harmonic oscillators • understand quasi-frequency, quasi-period and time constant • factor polynomials of degree more than two • use the characteristic equation to solve higher order DEs • be able to determine what DE a solution came from (work backwards)
4.4	Undetermined Coefficients	• know the superposition principle for nonhomogeneous linear DE and how to exploit it when solving equations • know how to apply the Nonhomogeneous Principle for nonhomogeneous linear equations • know when the method is applicable • utilize the method to solve appropriate nonhomogeneous second order DEs • predict forms of the particular solution based on the forcing function and solutions to the associated homogeneous equation
4.5	Variation of Parameters	• understand how the method is derived and when it is applicable • use the method to solve second order differential equations
4.6	Forced Oscillations	• solve DE associated with forced harmonic oscillators • determine the affect of damping on the mass/spring problem and observe the impact on solutions • understand resonance and beats in harmonic oscillators and their effects on solutions to the DE • determine a full solution to the damped, forced, mass-spring system • identify transient and steady state solutions • identify amplitude factor and interpret amplitude or frequency response curves

4.7	Conservation and Conversion	• recognize differential equations governing conservative systems • be able to compute total energy of conservative systems • know how to convert high order DE to first order systems
8.1	The Laplace Transform and Its Inverse	• understand how a transform is helpful in solving IVP • define the Laplace transform • know the properties of the Laplace transform • find inverse Laplace transforms
8.2	Solving DEs and IVPs with Laplace Transforms	• know the Laplace transform for the derivative of a function • know the translation and multiplication rules for Laplace transforms • use Laplace transforms to solve initial value problems
8.3	The Step Function and the Delta Function	• know the definition of the unit step function • use the step function to rewrite piecewise-defined functions • be able to relate piecewise-defined functions, their graphs and step functions • understand delayed functions and compute their Laplace transforms • know the definition of the delta function • use Laplace transforms to solve IVP involving discontinuous forcing functions
6.1	Theory of Linear DE Systems	• recognize a linear system of DE and associated IVP • understand what a solution to a linear system of DE is • apply the superposition and nonhomogeneous principles to systems of DE • apply the solution space theorem to linear homogeneous systems
6.2	Linear Systems with Real Eigenvalues	• understand how to find solutions to systems of first order DEs using eigenvalues/eigenvectors • provide the general solution for $n \times n$ systems of DE with distinct real eigenvalues • compute a generalized eigenvector and use it to find solutions of linear systems of DE with repeated real eigenvalues • interpret the behavior of solutions based on the eigenvalues • draw phase plane trajectories • create systems of linear DE from real world problems
6.3	Linear Systems with Nonreal Eigenvalues	• know how to build solutions when eigenvalues/eigenvectors are nonreal • classify the behavior of solutions with complex eigenvalues via the phase portrait

6.4	Stability and Linear Classification	• define fixed points and their relationship to equilibrium solutions • know the various geometries (center, node, spiral, saddle) of fixed points • be able to determine the stability of fixed points • distinguish asymptotic from neutral stability • be able to apply the trace/determinant plane • understand fast and slow eigendirections • know how nonisolated fixed points arise

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