APPM 2350 - Calculus 3 for Engineers

COURSE OBJECTIVES: Extend the ideas of single variable calculus to functions of several variables:

- three-dimensional space, vectors and vector-valued functions, functions of multiple variables, partial differentiation, multiple integrals
- vector analysis and the Fundamental Theorem of Line Integrals, Gauss, Green and Stokes

The class reinforces problem solving and critical thinking skills required for math, engineering and the sciences

TEXTBOOK:

- *Essential Calculus*, 2nd Edition by James Stewart, Chapter 10-13.
- Digital resources are also available through WebAssign
- Supplementary reading on Taylor Series is also available

PREREQUISITES:

- Any ONE of the following courses (minimum grade C-): APPM 1360 or MATH 2300

EQUIVALENT COURSES: Duplicate Degree Credit Not Granted:

- MATH 2400

SCHEDULE AND TOPICS COVERED

Day	Section / Pages	Topics
1	10.1 / 10.2	Three-Dimensional Coordinate Systems / Vectors [a]
2	10.2 / 10.3	Vectors [b] / The Dot Product [a]
3	10.3 / 10.4	The Dot Product [b] / The Cross Product [a]
4	10.4 / 10.5	The Cross Product [b] / Equations of Lines and Planes [a]
5	10.5 / 10.6	Equations of Lines and Planes [b] / Cylinders and Quadric Surfaces [a]
6	10.6 / 10.7	Cylinders and Quadric Surfaces [b] / Vector Functions and Space Curves [a]
7	10.7	Vector Functions and Space Curves [b]
8	10.8	Arc Length and Curvature [a]
9	10.8 / 10.9	Arc Length and Curvature [b] / Motion in Space: Velocity and Acceleration [a]
10	10.9 / 11.1	Motion in Space: Velocity and Acceleration [b] / Functions of Several Variables [a]
11	Review	Exam Review (Sections 10.1-10.9)
12	11.1 / 11.2	Functions of Several Variables [b] / Limits and Continuity [a]
13	11.2 / 11.3	Limits and Continuity [b] / Partial Derivatives [a]
14	11.3 / 11.4	Partial Derivatives [b] / Tangent Planes and Linear Approximations [a]
15	11.4 / Handout	Tangent Planes and Linear Approximations [b] / Taylor's Theorem for Functions of Two Variables [a]
16	Handout / 11.5	Taylor's Theorem for Functions of Two Variables [b] / The Chain Rule [a]
17	11.5 / 11.6	The Chain Rule [b] / Directional Derivatives and the Gradient Vector [a]
18	11.6 / 11.7	Directional Derivatives and the Gradient Vector [b] / Maximum and Minimum Values [a]
19	11.7 / 11.8	Maximum and Minimum Values [b] / Lagrange Multipliers [a]
20	11.8	Lagrange Multipliers [b]
21	12.1 / 12.2	Double Integrals over Rectangles / Double Integrals over General Regions [a]
22	12.2 / 12.3	Double Integrals over General Regions [b] / Double Integrals in Polar Coordinates [a]
23	Review	Exam Review (Sections 11.1-11.8, Handout)
24	12.3 / 12.4	Double Integrals in Polar Coordinates [b] / Applications of Double Integrals [a]
25	12.4 / 12.5	Applications of Double Integrals [b] / Triple Integrals [a]
26	12.5 / 12.6	Triple Integrals [b] / Triple Integrals in Cylindrical Coordinates [a]
27	12.6 / 12.7	Triple Integrals in Cylindrical Coordinates [b] / Triple Integrals in Spherical Coordinates [a]
28	12.7 / 12.8	Triple Integrals in Spherical Coordinates [b] / Change of Variables in Multiple Integrals [a]
29	12.8 / 13.A (pp 767-774, 803-810, 813-817)	Change of Variables in Multiple Integrals [b] / Scalar Line and Surface Integrals [a]
30	13.A (pp 767-774, 803-810, 813-817)	Scalar Line and Surface Integrals [b]

31	13.A (pp 767-774, 803-810, 813-817) / 13.B (pp 761-765, 795-799)	Scalar Line and Surface Integrals [c] / Vector Fields, Divergence and Curl [a]
32	13.B (pp 761-765, 795-799) / 13.C (pp 774-776, 817-822)	Vector Fields, Divergence and Curl [b] / Vector Line and Surface Integrals [a]
33	13.C (pp 774-776, 817-822) 13.D (pp 779-786)	Vector Line and Surface Integrals [b] / Fundamental Theorem for Line Integrals [a]
34	13.D (pp 779-786)	Fundamental Theorem for Line Integrals [b]
35	Review	Exam Review (Sections 12.1-13.B)
36	13.E (pp 788-793, 799-801)	Green's Theorem [a]
37	13.E (pp 788-793, 799-801)	Green's Theorem [b]
38	13.F (pp 824-828)	Stokes' Theorem [a]
39	13.F (pp 824-828)	Stokes' Theorem [b]
40	13.G (pp 829-834)	Gauss' Theorem [a]
41	13.G (pp 829-834)	Gauss' Theorem [b]
42	Review	Exam Review (Comprehensive)

Learning Objectives by Section

Section	Topics	Learning Objectives – After completing this section, students should be able to do the following:
10.1	Three-Dimensional Coordinate Systems	• represent points in three-dimensional space • know the three coordinate planes and various octants of three-dimensional space • know the right-hand rule for relating coordinate axes in three dimensions • understand the notation $\mathbb{R}, \mathbb{R}^2$, and $\mathbb{R}^3$ • derive and use the distance formula in three dimensions • give the equation for a sphere or ball and recognize the generalization from two dimensions.
10.2	Vectors	• state the definition of a vector • find vectors between points • relate position vectors to points in space • write vectors in component form • perform vector addition and scalar multiplication of vectors geometrically and algebraically • know the properties of vector addition and scalar multiplication • calculate the magnitude of a vector and find unit vectors • know the standard basis vectors and write vectors as linear combinations of the basis vectors • use vectors to solve applied problems

10.3	The Dot Product	• compute the dot product of two vectors using two different formulas • use dot products to compute the angle between vectors • know the relationship of dot products and orthogonal vectors • know properties of the dot product • find vector, scalar and orthogonal projections of vectors • use dot products to calculate work
10.4	The Cross Product	• compute the cross product of two vectors using two different formulas • understand symbolic determinants • know the relationships between vectors and their cross product • know the geometric meaning of the magnitude of the cross product of two vectors • know properties of the cross product • know the relationship of cross products and parallel vectors • define the scalar triple product and explain what it means geometrically • use cross products to calculate torque
10.5	Equations of Lines and Planes	• find the vector, parametric and symmetric equations of lines given information about the line • determine if two given lines are parallel, skew or intersecting • find the shortest distance from a point to a line using cross products • find the equation of a plane given information about the plane • given the equation of a plane, find a vector normal to the plane • find the shortest distance between a plane and a point • find the distance between two parallel planes • find the angle between two planes • find the line of intersection of two planes • find the point of intersection of a line and a plane
10.6	Cylinders and Quadric Surfaces	• understand the geometry of generalized cylinders • recognize equations that describe cylinders • know the geometry and equations of the quadric surfaces • define and sketch the quadric surfaces using traces • identify and sketch different traces for the quadric surfaces

10.7	Vector Functions and Space Curves	• define vector-valued functions and interpret them geometrically as a position vector • relate parametric curves to the space curve defined by a vector-valued function • find limits, derivatives, antiderivatives and definite integrals of vector-valued functions • relate the derivative of a vector-valued function to the vector tangent to the space curve • know the differentiation rules of vector-valued functions
10.8	Arclength and Curvature	• compute the arc length of a space curve • define the arc length function • parameterize a curve in terms of arc length • define and compute curvature of a space curve • interpret curvature in terms of the osculating circle • define, compute and understand the geometry of the unit tangent, normal and binormal vectors of a space curve • define the osculating, normal and rectifying planes associated with a space curve
10.9	Motion in Space: Velocity and Acceleration	• decompose the acceleration vector into tangential and normal components • understand the physics of tangential and normal acceleration • use vector-valued functions to analyze projectile motion and find forces on moving objects
11.1	Functions of Several Variables	• define a function of several variables • find the domain and range of a function of several variables • visualize graphs of functions of two variables • understand traces and level curves of functions of two variables • create contour plots (maps) of functions of two variables • generate level surfaces for functions of three variables
11.2	Limits and Continuity	• informally explain the concept of the limit of a function of two variables • apply theorems that guarantee limits of multivariable functions exist • understand the two path criterion to show that a limit does not exist and apply it to solve problems about limits • determine continuity of functions of several variables
11.3	Partial Derivatives	• calculate partial derivatives of multivariable functions • recognize various notation for partial derivatives • provide geometrical meaning of partial derivatives • estimate partial derivatives from tables • estimate partial derivatives contour plots • determine if a function satisfies a given partial differential equation

11.4	Tangent Planes and Linear Approximations; Taylor's Theorem for Functions of Two Variables (supplemental reading)	• find explicit formulas for tangent planes • understand the relationship between linear approximation and the tangent plane • understand the definition of differentiability for multivariable functions • find linear approximations of multivariable functions • use linearizations to estimate values of functions • calculate differentials of functions of multiple variables • use differentials to approximate changes in multivariable functions • use differentials in applied problems • compute Taylor series of functions of two variables • compute Taylor polynomials of functions of two variables and analyze the errors in the approximations
11.5	The Chain Rule	• define the chain rules for functions of more than one variable • compute the derivative of the composition of a function of several variables and a vector-valued function and describe its applications • compute the derivatives of the composition of a function of several variables, each of which is a function of several variables • use the multivariable chain rule to perform implicit differentiation
11.6	Directional Derivatives and the Gradient Vector	• compute directional derivatives • interpret directional derivatives graphically • estimate directional derivatives from contour maps • compute gradient vectors and understand their relationship to level curves and level surfaces • understand rates of change of multivariable functions and their relationship to gradient vectors • know the relationship between rates of change with respect to time and distance
11.7	Maximum and Minimum Values	• define and find the critical points of a function of two variables • recognize relative maxima, minima and saddle points of functions of two variables using graphs and level curves • classify critical points of functions of two variables using the Second Derivatives Test • solve applied optimization problems involving functions of two variables • know and apply the Extreme Value Theorem for multivariable functions
11.8	Lagrange Multipliers	• understand the geometric basis of the method of Lagrange multipliers • use Lagrange multipliers to solve constrained optimization problems

12.1	Double Integrals over Rectangles	• understand the Riemann sum definition of a double integral, both algebraically and geometrically • know the geometric meaning of a double integral • use iterated integrals to compute double integrals over rectangular regions • apply Fubini's Theorem
12.2	Double Integrals over General Regions	• compute definite integrals over general domains • apply Fubini's theorem to general domains of integration • use double integrals to calculate areas and volumes • sketch the region of integration given a double integral • change the order of integration in a double integral
12.3	Double Integrals in Polar Coordinates	• understand algebraically and geometrically how areas in Cartesian coordinates are related to areas in polar coordinates • compute double integrals in polar coordinates • convert polar coordinate double integrals to Cartesian coordinate double integrals and vice versa
12.4	Applications of Double Integrals	• use double integrals to calculate average value • use double integrals to calculate mass, moments, center of mass, moments of inertia of two-dimensional objects
12.5	Triple Integrals	• understand the Riemann sum definition of a triple integral both algebraically and geometrically • use iterated integrals to compute triple integrals • apply Fubini's theorem to triple integrals • use triple integrals to calculate volume, mass, and center of mass of three-dimensional solids • use triple integrals to calculate average values • change the order of integration in triple integrals
12.6	Triple Integrals in Cylindrical Coordinates	• understand both algebraically and geometrically how volumes in cylindrical coordinates are related to volumes in Cartesian coordinates • convert triple integrals from Cartesian to cylindrical coordinates and vice versa • calculate volume, mass, center of mass and average values using cylindrical coordinates • change the order of integration in cylindrical coordinates • draw regions in space given a triple integral in cylindrical coordinates

12.7	Triple Integrals in Spherical Coordinates	• understand both algebraically and geometrically how volumes in spherical coordinates are related to volumes in Cartesian coordinates • convert triple integrals from Cartesian to spherical coordinates and vice versa • convert triple integrals from cylindrical to spherical coordinates and vice versa • calculate volume, mass, center of mass and average values using spherical coordinates • change the order of integration in spherical coordinates • draw regions in space given a triple integral in spherical coordinates
12.8	Change of Variables in Multiple Integrals	• understand the need to use change of variables in multiple integrals • know the basics of transformations in two- and three-dimensional space • compute Jacobians of transformations and understand their geometric meaning • identify transformations to use in multiple integral problems • apply the Change of Variables Theorem
13.A	Scalar Line and Surface Integrals	• know the definition and different forms of scalar line integrals • parameterize paths to evaluate scalar line integrals • use scalar line integrals to compute area, arclength of curves, average values, mass, moments and centers of mass • know the definition and different forms of scalar surface integrals • parameterize surfaces to evaluate scalar surface integrals • use scalar surface integrals to compute surface area, average values, mass, moments and centers of mass
13.B	Vector Fields, Divergence and Curl	• explain the concept of a vector field and make sketches of simple vector fields in the plane • define gradient vector fields • know what a conservative vector field is and its relationship to a potential function • compute the divergence of a vector field and describe its physical meaning • compute the curl of a vector field and describe its physical meaning
13.C	Vector Line and Surface Integrals	• know the definition and different forms of vector line integrals • parameterize paths to evaluate vector line integrals • use vector line integrals to compute work, flow, and circulation along two-or three-dimensional curves, and flux through two-dimensional curves • know the definition and different notation used for vector surface integrals • parameterize surfaces to evaluate vector surface integrals • use vector surface integrals to compute the flux of a vector field through a surface • understand the physical meanings of flux and flow

13.D	Fundamental Theorem for Line Integrals	• know the necessary and sufficient conditions required for a vector field to be conservative • find potential functions for conservative vector fields • define path independence • state and apply the Fundamental Theorem for Line Integrals
13.E	Green's Theorem	• state Green's Theorem in both its flow and flux form and describe the physical meanings of the various integrals • use Green's Theorem to compute areas • know when to use the correct form of Green's Theorem to compute various integrals and quantities defined as integrals
13.F	Stokes' Theorem	• state Stokes' Theorem and describe the physical meaning of the various integrals • interpret Stokes' Theorem as a three-dimensional version of Green's Theorem • know how to use Stokes' Theorem to compute various integrals and quantities defined as integrals
13.G	Gauss' Divergence Theorem	• state Gauss' Divergence Theorem and describe the physical meaning of the various integrals • interpret Gauss' Divergence Theorem as a three-dimensional version of Green's Theorem • know how to use Gauss' Theorem to compute various integrals and quantities defined as integrals

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