This exam is worth 125 points and has **problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **match each problem with your work in Gradescope**.

Show all work and simplify your answers. Be sure that your work is legible and organized.

Answers with no justification will receive no points unless the problem explicitly states otherwise.

Name any theorem you use and explain how it is used.

Turn in your paper exam before you leave the room.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

Honor Code Pledge

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this exam.

At the top of the first page of your test, please write "I will abide by the Honor Code Pledge on this exam." and sign your name.

Formulas

$$\sin(2x) = 2\sin(x)\cos(x) \qquad \sin^2(x) = \tfrac{1}{2}\left(1 - \cos(2x)\right)$$
$$\cos(2x) = \cos^2(x) - \sin^2(x) \qquad \cos^2(x) = \tfrac{1}{2}\left(1 + \cos(2x)\right)$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$$

1. (**30 pts**) Evaluate the following. You may use any calc 1 techniques.

   (a) Evaluate $\lim_{x\to 0} \dfrac{xe^x}{e^x - 1}$

   (b) Evaluate $\lim_{x\to \frac{\pi}{2}^-} \dfrac{\sec x}{\tan x}$

   (c) Evaluate $\lim_{x\to\infty} \left(1 + \dfrac{2}{x}\right)^x$

2. (**15 pts**) A box with a square base and open top must have a volume of 32,000cm^3. Find the dimensions of the box that minimize the amount of material used.

3. (**26 pts**) For the following problems, consider the decreasing function $f(x) = \dfrac{-\sqrt{e^x}}{2}$.

   (a) Find the domain and range of $f(x)$.

   (b) Find $f^{-1}(x)$. Indicate the domain and range of the inverse function.

   (c) Find the equation of the tangent line to $f^{-1}(x)$ when $x = \dfrac{-1}{2}$.

4. (**36 pts**) The following problems are unrelated. You may use any calc 1 techniques.

   (a) Compute $\displaystyle\int \frac{x}{\sqrt{16x^2+1}}\,\mathrm{d}x$

   (b) Find $\dfrac{dy}{dx}$ if $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$ Hint: use logarithmic differentiation.

   (c) Evaluate $\displaystyle\int_{1/2}^{1/\sqrt{2}} \frac{4}{\sqrt{1-x^2}}\,\mathrm{d}x$

5. (**18 pts**) Consider a bacteria culture that grows at a rate proportional to its size. The size of the culture increases by 50% in one hour. Let $P(t)$ be the number of bacteria after t hours.

   (a) Find the relative growth rate of the culture (k).

   (b) Find an expression for $P(t)$. Fully simplify your answer.

   (c) If the number of cells in the culture is 900 after two hours, what was the initial population of the culture?

END OF TEST