Answer the following problems, showing all of your work and simplifying your solutions where possible unless otherwise stated.

No calculators, notes, books, electronic devices, internet access, AI tools etc. are allowed. This is a closed book, closed note exam.

1. (22 pts) Evaluate the integrals.

   (a) $\int \frac{2x^2}{\sqrt{4-x^2}}\mathrm{d}x$

   (b) $\int_{-2}^{14} \frac{1}{\sqrt[4]{x+2}}\mathrm{d}x$

2. (46 pts) Let

$$f(x) = \ln(x).$$

   (a) Find a Taylor series centered about $a = 1$ for $f(x)$.

   (b) What is the radius and interval of convergence for the series in part a?

   (c) Use $T_2(x)$ to estimate $f(1.1)$.

   (d) Use Taylor's Remainder Formula to find an upper bound on the error in your estimation from part c.

3. (26 pts) The following questions are related.

   (a) Find a Maclaurin series for $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2}$.

   (b) Find a Maclaurin series for $\int \frac{1}{\sqrt{2\pi}}e^{-x^2}\mathrm{d}x$. Include the radius of convergence.

   (c) Use your answer in part a to evaluate $\int_0^{1/4} \frac{1}{\sqrt{2\pi}}e^{-x^2}dx$ exactly. (Hint: your solution will be in the form of a series).

4. (20 pts) Consider the following parametric equations:

$$x = -\ln(t) \quad y = -\frac{1}{2}t^2 + 1 \quad t > 0$$

   (a) Find the equation of the line tangent to the curve at the point $(0, \frac{1}{2})$.

   (b) Eliminate the parameter t to find an equation for y in terms of x.

5. (36 points) Consider the polar equation $r^2 = \cos^2\theta$.

   (a) Sketch the polar curve.

   (b) **Evaluate an integral** to find the total area inside the curve.

   (c) **Evaluate an integral** to find the length of the curve.

Trigonometric Identities

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$ $\sin(2x) = 2\sin(x)\cos(x)$ $\cos(2x) = \cos^2(x)-\sin^2(x)$

Inverse Trigonometric Integral Identities

$$\int \frac{\mathrm{d}u}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C,\ u^2 < a^2$$
$$\int \frac{\mathrm{d}u}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{\mathrm{d}u}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C,\ u^2 > a^2$$

Common Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)x^2}{2!} + \frac{k(k-1)(k-2)x^3}{3!} + \cdots \qquad R = 1$$