- This exam is worth 150 points and has 7 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on two sides.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the page containing your solution to problem 1, write the following statement and sign your name to it: "On my honor, I confirm the work herein is mine and has not been created using any computer resources." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/072525 (18 pts)] Let $\mathbf{F} = e^y\,\mathbf{i} + (xe^y + \sin z)\,\mathbf{j} + y\cos z\,\mathbf{k}$.

   (a) (6 pts) Show that $\mathbf{F}$ is conservative.

   (b) (12 pts) Find the work done by $\mathbf{F}$ on an object that moves from $(0,0,0)$ to $(1,-1,3)$.

2. [2350/072525 (16 pts)] Consider the function $f(x,y) = x + xy - y$. For the following questions, you don't need to find any actual values, simply justify your answers.

   (a) (8 pts) Does $f(x,y)$ possess any local (relative) extreme values on $\mathbb{R}^2$?

   (b) (8 pts) Does $f(x,y)$ possess any extreme values (local/relative or global/absolute) on the region $|x-1| \le 1,\ |y+1| \le 1$?

3. [2350/072525 (20 pts)] Let $\mathcal{S}$ be the first octant portion of the plane with intercepts $(2,0,0)$, $(0,4,0)$ and $(0,0,1)$. Its surface area is $\sqrt{21}$. Using this information, find the average value of $f(x,y,z) = 1 + x$ on $\mathcal{S}$.

4. [2350/072525 (18 pts)] Use Green's Theorem to find the outward flux of the vector field $\mathbf{F} = x^2y\,\mathbf{i} + 3xy^2\,\mathbf{j}$ through the boundary of the second quadrant portion of the circle of radius 3 centered at the origin. No points awarded if Green's Theorem is not used.

5. [2350/072525 (20 pts)] Use Gauss' Divergence Theorem to find the outward flux of the vector field $\mathbf{F} = x^3\,\mathbf{i} + y^3\,\mathbf{j} + z^3\,\mathbf{k}$ through the boundary of the solid region above the cone $z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 4$. No points awarded if Gauss' Divergence Theorem is not used.

6. [2350/072525 (18 pts)] A piece of wire is in the shape of (e^t, t^2) with its left end at the point $(1,0)$. The charge density on the wire is $q(x,y) = \dfrac{3y}{\sqrt{x^2+4y}}$ Coulombs per meter. If the total charge on the wire is 8 Coulombs, find the coordinates of the right end of the wire.

7. [2350/072525 (40 pts)] Let $\mathbf{V} = y\,\mathbf{i} + yz\,\mathbf{j} - \frac{1}{2}x^2\,\mathbf{k}$ be the velocity of a fluid and consider the surface, $\mathcal{S}$, given by $x^2+y^2-z^2 = -1$, $1 \le z \le \sqrt{5}$ with upward pointing normal.

   (a) (5 pts) Name the surface.

   (b) (15 pts) Find the circulation of $\mathbf{V}$ on the boundary of $\mathcal{S}$ by direct computation. The identity $1 - 2\sin^2 t = \cos 2t$ may be helpful.

   (c) (20 pts) Find the circulation of $\mathbf{V}$ on the boundary of $\mathcal{S}$ using Stokes' Theorem. No points awarded if Stokes' Theorem not used.