

Answer the following problems, showing all of your work and simplifying your solutions where possible unless otherwise stated.

No calculators, notes, books, electronic devices, internet access, AI tools etc. are allowed. This is a closed book, closed note exam.

1. (30 pts) Are the following series convergent or divergent? Justify your answer and state which test was used.

(a) (10 pts) $\sum_{m=1}^{\infty} \frac{m}{m^4 + 2}$

(b) (10 pts) $\sum_{k=2}^{\infty} \frac{[(k+1)!]^2}{(k^2 - 1)(k!)^2}$

(c) (10 pts) $\sum_{n=3}^{\infty} \frac{\ln(n^3)}{2n}$

2. (34 pts) Consider the following series, which sums to S :

$$S = \sum_{n=1}^{\infty} \frac{(-3)^n}{4^n(n!)}$$

- (a) (6 pts) Write out s_1, s_2 , and s_3 , the first three partial sums. You do not need to simplify your answer.
- (b) (12 pts) State the hypotheses of the Alternating Series Estimation Theorem and show that each is satisfied.
- (c) (6 pts) Find an estimate for the error if s_3 is used to approximate S . You do not need to simplify your answer.
- (d) (10 pts) Does this series converge absolutely or conditionally?

3. (10 pts) Find the sum of the series, if it exists, for $\sum_{n=1}^{\infty} \frac{1 + 3^{n-1}}{5^n}$.

4. (10 pts) Consider the following power series:

$$\sum_{n=0}^{\infty} c_n (x+1)^n$$

This power series is convergent for $x = -2$, and divergent for $x = 5$.

Are the following statements about this power series true or false? Justify your answer.

- (a) The series definitely diverges for $x = -7$.
- (b) We need more information to say whether $\sum_{n=0}^{\infty} c_n$ diverges.

5. (16 pts) Find a power series representation for the following function:

$$f(x) = \frac{8}{8 + x^3}$$

Include the interval and radius of convergence for the power series.