This exam is worth 100 points and has **5 problems**.

Do your work on blank paper. **Begin each problem on a new page.** Be sure to **match each problem with your work in Gradescope**.

Clearly write your name and the problem number at the top of each page.

Show all work and simplify your answers. Be sure that your work is legible and organized.

Answers with no justification will receive no points unless the problem explicitly states otherwise.

Name any theorem you use and explain how it is used.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

Notify your proctor **before starting to scan** and upload your exam.

Honor Code Pledge

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this exam.

At the top of the first page of your test, please write "I will abide by the Honor Code Pledge on this exam." and sign your name.

Double-Angle Identities

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\begin{aligned}\cos(2x) &= \cos^2(x) - \sin^2(x)\\ &= 1 - 2\sin^2(x)\\ &= 2\cos^2(x) - 1\end{aligned}$$

Summation Properties:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

1. (**22 pts**) Consider $g(x) = \displaystyle\int_{-1}^{-\sqrt{x+1}} \frac{1}{t^2+3t}\,\mathrm{d}t$

   (a) Find $g'(x)$

   (b) Find the equation of the tangent line to $g(x)$ when $x = 0$.

2. (**28 pts**) Consider $\displaystyle\int_0^{\pi} \cos^2(x)\,\mathrm{d}x$

   (a) Compute R_3, the right endpoint approximation using 3 equispaced intervals.

   (b) Write the integral as a limit of Riemann sums (do not evaluate the limit).

   (c) Evaluate the integral. (Hint: you may want to use a trig identity).

3. (**15 pts**) The acceleration of gravity on Mars is about $\frac{15}{4}\ m/s^2$. The height of Olympus Rupes, the cliff surrounding Olympus Mons (the largest volcano in our solar system) can reach 7,500 m. Suppose an astronaut drops a rock off the highest point on the Olympus Rupes cliff. Answer the following questions.

   (a) How long will it take for the rock to hit the ground?

   (b) What is the velocity of the rock when it hits the ground?

4. (**15 pts**) Consider $f(x) = x^3 + 5x^2 - 1$.

   (a) Show that Newton's method will fail to converge to a root of this function with an initial guess of $x_0 = \dfrac{-10}{3}$.

   (b) Find x_1 if $x_0 = -1$ (first iteration of Newton's method given an initial guess).

5. (**20 pts**) Evaluate the following:

   (a) $\displaystyle\int_0^2 |x-2| - \sqrt{4-x^2}\,\mathrm{d}x$. Hint: use geometry.

   (b) $\displaystyle\int \sqrt{x}\sin\left(1+\sqrt{x^3}\right)\mathrm{d}x$

END OF TEST