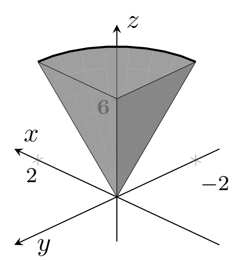
- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/071125 (15 pts)] Let $\mathcal{R}$ be the parallelogram in the xy-plane enclosed by the lines $x+4y=4, x+4y=9, x-y=1$ and $x-y=4$. Use a change of variables to find the volume of the solid above the region $\mathcal{R}$ and below the surface $z=5\sqrt{(x-y)(x+4y)}$.

2. [2350/071125 (15 pts)] A metal pipe with inner diameter d_i, outer diameter d_o and length l has a mass density that varies inversely with the cube of the distance from the axis of the pipe, that is, mass density, $\delta = k/\text{distance}^3$. Find the constant of proportionality, k, in terms of the other variables, if the total mass of the pipe is M. (Recall that the total mass of a three-dimensional solid is given by the triple integral of the mass density over the region occupied by the solid.) Hint: Place the axis of the pipe along the z-axis and pick a coordinate system that simplifies the problem.

3. [2350/071125 (18 pts)] Consider the solid, $\mathcal{W}$, shown below, which is a portion of the cone $x^2+y^2-\left(\frac{z}{3}\right)^2=0$.



Each of the following triple integrals can be used to compute the volume of $\mathcal{W}$. Copy each them onto your paper and provide the six (6) appropriate limits for each one, using the given order of integration. **Do not evaluate** any of the integrals. To receive full credit, you must use the correct bounds for the figure as shown (study it carefully), not bounds for an equivalent solid in a different octant.

(a) Volume ($\mathcal{W}$) = $\int\int\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

(b) Volume ($\mathcal{W}$) = $\int\int\int r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

(c) Volume ($\mathcal{W}$) = $\int\int\int \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

4. [2350/071125 (15 pts)] Using spherical coordinates with integration order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$, set up, but **do not evaluate**,

$$\iiint_Q \frac{z}{\sqrt{1+x^2+y^2}}\,\mathrm{d}V,$$

where Q is the region of the sphere $x^2+y^2+z^2=4$ below the plane $z=-\sqrt{3}$ and under the first octant.

MORE PROBLEMS BELOW/ON REVERSE

5. [2350/071125 (22 pts)] Use polar coordinates to combine the following into a single double integral and then evaluate the resulting polar coordinate double integral. Making a sketch should prove beneficial.

$$\int_{1/\sqrt{2}}^{1} \int_{\sqrt{1-x^2}}^{x} xy \, \mathrm{d}y \, \mathrm{d}x + \int_{1}^{\sqrt{2}} \int_{0}^{x} xy \, \mathrm{d}y \, \mathrm{d}x + \int_{\sqrt{2}}^{2} \int_{0}^{\sqrt{4-x^2}} xy \, \mathrm{d}y \, \mathrm{d}x$$

6. [2350/071125 (15 pts)] Evaluate $\displaystyle\int_0^4 \int_0^1 \int_{2y}^{2} \frac{2\cos(x^2)}{\sqrt{z}} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$. Hint: The antiderivative of $\cos(x^2)$ is not $-\sin\left(x^2\right)$.