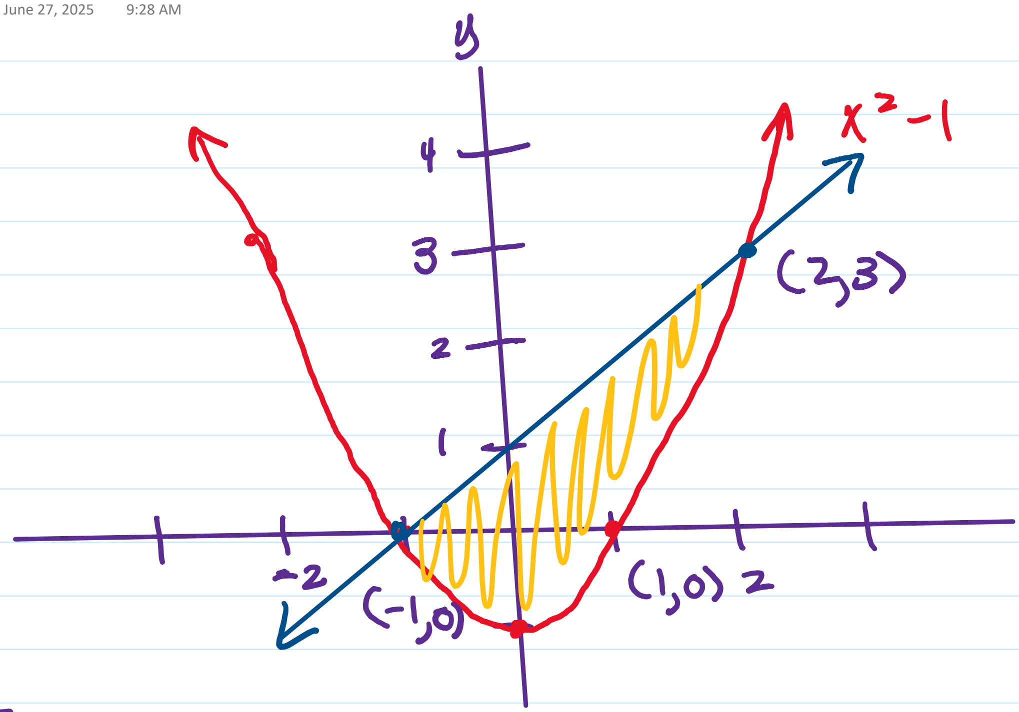

Intersects:

$$x^2-1=0 \quad @ \quad x=1,\ x=-1$$

$$x^2-1=x+1$$
$$x^2-x-2=(x-2)(x+1)$$
$$x=2 \qquad x=-1$$

b) Revolved about $y=3$, we use washers.

$$R=3-(x^2-1)$$
$$r=3-(x+1)$$

$$\int_{-1}^{2}\pi\left[(3-(x^2-1))^2-(3-(x+1))^2\right]dx$$

c) We use shells.

$$r=x-(-1)=x+1$$
$$h=[(x+1)-(x^2-1)]$$

$r = x - (-1) = x + 1$

$h = [(x+1) - (x^2 - 1)]$

$$\int_{-1}^{2} 2\pi (x+1)[(x+1) - (x^2-1)]\,dx$$

2) a) $dy/dx = \left(\frac{1}{2}x - \frac{1}{2x}\right)$

$$ds = \sqrt{1 + \left(\frac{1}{2}x - \frac{1}{2x}\right)^2}\,dx$$

$$= \sqrt{1 + \left(\frac{1}{4}x^2 - \frac{1}{4} - \frac{1}{4} + \frac{1}{4x^2}\right)}\,dx$$

$$= \sqrt{\left(\frac{1}{2}x + \frac{1}{2x}\right)^2}\,dx$$

$$= \left(\frac{1}{2}x + \frac{1}{2x}\right)$$

Then: $\displaystyle \text{Arc} = \int_1^e \frac{1}{2}x + \frac{1}{2x}\,dx$

$$= \frac{1}{4}x^2 + \frac{1}{2}\ln(x)\Big|_1^e$$

$$= \frac{1}{4}e^2 + \frac{1}{2} - \frac{1}{4} = \frac{1}{4}(e^2+1)$$

4) a)

This seq evaluates to

$\{1, 0, -1, 0, 1, 0, \ldots\}$

and oscillates forever. thus diverges

b) $c_n = \frac{1}{4}(n-1)!(-n)^{-n} = \frac{1}{4}(-1)^n \left(\frac{(n-1)!}{n^n}\right)$

if $\left(\frac{(n-1)!}{n^n}\right)$ goes to 0, then $(-1)^n \cdot b_n$ does.

This is always > 0, and

$$\frac{(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1}{n\cdot n\cdot n\cdots n} = \frac{1}{n}\left(\frac{(n-1)!}{n^{n-1}}\right)$$

$(n-1)! < n^{n-1}$, so this is $< \frac{1}{n}\cdot 1$

and

$$c \lim_{n\to\infty} a_n = \lim_{n\to\infty} c a_n$$

so we consider w/out the constant.

Then:

$$0 \le \frac{(n-1)!}{n^n} \le \frac{1}{n}$$

$\lim_{n\to\infty} 0 = 0$ and $\lim_{n\to\infty} \frac{1}{n} = 0$

thus

$$0 \le \lim_{n\to\infty} \frac{(n-1)!}{n^n} \le 0$$

By squeeze thm, $b_n \to 0$ as $n \to \infty$

Then

$$(-1)^n b_n \to 0 \quad \text{as} \quad n \to \infty$$

So

$$\lim_{n\to\infty} \frac{1}{4} \frac{(n-1)!}{n^n} \cdot (-1)^n = 0$$

c) Consider continuous analogue

$$\lim_{x\to\infty} \frac{7x}{\ln(x^2)}$$ this is $\frac{\infty}{\infty}$ indet

So

l'H:

$$\lim_{x\to\infty} \frac{7}{\frac{2x}{x^2}} = \frac{7x}{2} = \infty$$

this diverges.

#2b, revolve about x-axis

$$\int_1^e 2\pi\left(\frac{1}{4}x^2 - \frac{1}{2}\ln(x)\right)\left(\frac{x}{2}+\frac{1}{2x}\right)dx$$

$$= 2\pi\int_1^e \frac{1}{8}x^3 - \frac{1}{4}x\ln(x) + \frac{1}{8}x - \frac{1}{4}\frac{\ln(x)}{x}\,dx$$

$$= 2\pi\left[\frac{1}{8}\int_1^e x^3 + x\,dx - \frac{1}{2}\int_1^e \frac{\ln(x)}{x}\,dx - \frac{1}{4}\int_1^e x\ln(x)\,dx\right]$$

1st integral

$$= \frac{1}{8}\cdot\left(\frac{1}{4}x^4 + \frac{1}{2}x^2\right)\Big|_1^e$$

$$= \frac{e^4}{32} + \frac{e^2}{16} - \frac{3}{32}$$

2nd integral

$$-\frac{1}{4}\int_1^e \frac{\ln(x)}{x}\,dx$$

$u = \ln(x)$ $du = \frac{1}{x}dx$
when $x=e$, $u=1$
when $x=1$, $u=0$

$$= -\frac{1}{4}\int_0^1 u\,du$$

$$= -\frac{1}{8}u^2\Big|_0^1$$

$$= -\frac{1}{8} \Big| -\frac{4}{82}$$

3rd integral:

let $u = \ln(x)$, $du = \frac{1}{x}\,dx$,

$dv = x\,dx$, $v = \frac{1}{2}x^2$

$$-\frac{1}{4}\int_1^e x\ln(x)\,dx = -\frac{1}{4}\left[\frac{x^2\ln(x)}{2}\Big|_1^e - \frac{1}{2}\int_1^e \frac{x^2}{x}\,dx\right]$$

$$= -\frac{1}{4}\left[\frac{e^2}{2} - \left(\frac{1}{4}x^2\Big|_1^e\right)\right]$$

$$= -\frac{1}{4}\left[\frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4}\right]$$

$$= -\frac{e^2}{8} + \frac{e^2}{16} - \frac{1}{16} = -\frac{e^2}{16} - \frac{1}{16}$$

$$\text{ans} = 2\pi\left[\frac{e^4}{32} + \cancel{\frac{e^2}{16}} - \frac{3}{32} - \frac{4}{32} - \frac{2}{32} - \cancel{\frac{e^2}{16}}\right]$$

$$= \frac{\pi}{16}\left[e^4 - 9\right]$$

Thursday, June 26, 2025 10:31 PM

$$\frac{dy}{dt} = 3y(1 - y/2)$$

$$\int \frac{1}{3y(1-y/2)}\,dy = \int 1\,dt$$

$$\frac{1}{3y(1-y/2)} = \frac{A}{3y} + \frac{B}{(1-y/2)}$$

$$1 = A(1 - y/2) + 3By$$

$$y = 0 \Rightarrow A = 1$$

$$y = 2 \Rightarrow$$

$$1 = 6B$$

$$B = 1/6$$

$$\int \frac{1}{3y} + \frac{1}{6(1-y/2)}\, dy = t + C$$

$$u = 1 - y/2 \quad du = -1/2\, dy$$

$$\frac{1}{3}\ln(y) - \frac{1}{3}\ln|1 - y/2| = t + C$$

$$\ln|y| - \ln|1 - y/2| = 3t + C_2$$

$$\left|\frac{y}{1-y/2}\right| = e^{3t + C_2}$$

$$\frac{y}{1 - y/2} = Ae^{3t}$$

$$y = (1 - y/2)Ae^{3t}$$

$$y = Ae^{3t} - \frac{y}{2}Ae^{3t}$$

$$y + \frac{y}{2}Ae^{3t} = Ae^{3t}$$

$$y\left(1 + \frac{1}{2}Ae^{3t}\right) = Ae^{3t}$$

$$u = Ae^{3t}$$

$$y = \frac{Ae^{3t}}{1+\frac{1}{2}Ae^{3t}}$$

$$y(0) = 2/3$$

$$2/3 = \frac{A}{1+\frac{1}{2}A}$$

$$\frac{2}{3} = \frac{A}{\frac{2+A}{2}}$$

$$\frac{2}{3} = \frac{2A}{2+A}$$

$$\Rightarrow A = 1$$

$$y = \frac{e^{3t}}{1+\frac{1}{2}e^{3t}}$$

$$\lim_{n\to\infty} \sqrt{\frac{cn+d}{16n+1}} \cdot \frac{\frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{c+\frac{d}{n}}}{\sqrt{16+\frac{1}{n}}}$$

$$\lim_{n \to \infty} a_n = \frac{\sqrt{c}}{4}$$

$c = 4$

$d =$ any number