Answer the following problems, showing all of your work and simplifying your solutions where possible.

No calculators, notes, books, internet access etc. are allowed. This is a closed book, closed note exam.

1. (20 pts) Let $\mathcal{R}$ be the region bounded by the curves $x^2 - 1$ and $x + 1$. Do not simplify your integrands.
   (a) (4 pts) Sketch the region, labeling all axes, intersects, and curves.
   (b) (8 pts) Set up, **but do not evaluate,** an integral to find the volume of the solid generated when $\mathcal{R}$ is revolved around the line $y = 3$.
   (c) (8 pts) Set up, **but do not evaluate,** an integral to find the volume of the solid generated when $\mathcal{R}$ is revolved around the line $x = -1$.

2. (32 points) Consider the curve $\frac{1}{4}x^2 - \frac{1}{2}\ln(x)$ on the interval $1 \le x \le e$.
   (a) (12 pts) Find the arc length of this curve.
   (b) (20 pts) The curve is rotated about the x axis to create a surface. Find the surface area.

3. (20 pts) Solve the following initial value problem. Give the solution in explicit form.

$$\frac{dy}{dt} = 3y(1 - \frac{y}{2}),\ y(0) = \frac{2}{3}$$

4. (21 pts) Do the following sequences converge or diverge? Justify your answer. If they converge, find the limit of the sequence.
   (a) (7 pts) $\{\sin(\frac{\pi}{2}), \sin(\pi), \sin(\frac{3\pi}{2}), ..., \sin(\frac{n\pi}{2}), ...\}$
   (b) (7 pts) $c_n = \frac{1}{4}(n-1)!(-n)^{-n}$
   (c) (7 pts) $\left\{\frac{7n}{\ln(n^2)}\right\}_{n=1}^{\infty}$

5. (7 pts) For what values c and d, with $c, d > 0$, does

$$a_n = \sqrt{\frac{cn + d}{16n + 1}}$$

converge to $\frac{1}{2}$? Justify your answer.

Trigonometric Identities

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ $\quad \sin^2(x) = \frac{1}{2}(1-\cos(2x))$ $\quad \sin(2x) = 2\sin(x)\cos(x)$ $\quad \cos(2x) = \cos^2(x)-\sin^2(x)$

Inverse Trigonometric Integral Identities

$$\int \frac{\mathrm{d}u}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C,\ u^2 < a^2$$
$$\int \frac{\mathrm{d}u}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{\mathrm{d}u}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C,\ u^2 > a^2$$

Center of Mass/Centroids

$$m = \int_a^b \rho(f(x)-g(x))\mathrm{d}x$$
$$M_y = \int_a^b \rho x(f(x)-g(x))\mathrm{d}x$$
$$M_x = \int_a^b \frac{\rho}{2}\left[(f(x))^2-(g(x))^2\right]\mathrm{d}x$$
$$\overline{x} = \frac{M_y}{m} \text{ and } \overline{y} = \frac{M_x}{m}$$