

1. (20 pts) Consider the equation $y^2 - xy = 3\sqrt{x} - 1$.
 - (a) Find $\frac{dy}{dx}$.
 - (b) Find the equation of the tangent line to the curve when $x = 4$ and $y > 0$.
2. (15 pts) The top of a ladder slides down a vertical wall at a rate of 1/3 ft/s. At the moment when the bottom of the ladder is 5ft from the wall, it is sliding along the ground at a rate of 4/5 ft/s.
 - (a) How high is the top of the ladder from the ground at the instant that the base is 5ft from the wall?
 - (b) How long is the ladder?
3. (40 pts) Consider $f(x) = \cos x + \frac{x}{2}$ on the interval $[0, 2\pi]$.
 - (a) Find the intervals over which f is increasing and decreasing.
 - (b) Find and classify all local extrema (coordinate pairs) if any.
 - (c) Find the absolute maximum and minimum values attained by the function on $[0, 2\pi]$.
 - (d) Find the intervals of concavity of f .
 - (e) Find all points of inflection (coordinate pairs) if any.
 - (f) Sketch a graph of $y = f(x)$. Be sure to label any key points including extrema and points of inflection.
4. (25 pts) Consider $g(x) = \frac{1}{(x+2)^k}$ for some $k > 0$.
 - (a)
 - i. Find the linearization of g centered at $a = 1$.
 - ii. Use your linearization to estimate the value of $\frac{1}{3.01^3}$.
 - iii. Do you expect your estimate to be greater or less than the true value of $\frac{1}{3.01^3}$? Justify your answer.
 - (b) Let $k = 4$. Show that there is no value of c that satisfies the conclusion of the Mean Value Theorem on $[-4, 0]$. Explain, using the hypotheses, why this is not a contradiction of MVT.