- This exam is worth 100 points and has 8 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5”$\times$ 11” crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

---

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: ”I will abide by the CU Boulder Honor Code on this exam.” **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/062725 (15 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   Consider the function $z = f(x, y) = 25x^2 + 4y^2 + 4$.

   (a) The level curves of $f(x, y)$ are circles.

   (b) The tangent plane to the function at $(x, y) = (0, 0)$ is horizontal.

   (c) The vertical trace of the function in the plane $y = 2$ is an ellipse.

   (d) If you were to walk along the curve $(2\cos t, 5\sin t)$ in the xy-plane, the height of the surface above you would be constant.

   (e) The domain and range of the function are, respectively, $\mathbb{R}^2$ and $[0, \infty)$.

2. [2350/062725 (12 pts)] You are making a rectangular chicken run using 12 feet of fencing for a boundary. Use Lagrange Multipliers to find the dimensions of the run that will give the chickens the most area in which to frolic.

3. [2350/062725 (18 pts)] A certain portion of a forest consists of its boundary, given by the lines $|x| = 5$, $|y| = 5$, and the region inside the boundary. The elevation in the forest is $h(x, y) = 3xy - x^3 - y^3 + 2$.

   (a) (4 pts) A friend of yours (as well as your grader) does not want numbers or lengthy calculations, just a simple yes or no answer, with a verbal mathematical justification, to the question “Is there a highest and lowest point in the forest?”

   (b) (8 pts) Are there any saddles (passes) or local hills or valleys inside the boundaries of the forest? If so, find their locations and their elevations. If there are none, explain why not.

   (c) (6 pts) You and your friend are hiking along a trail whose projection onto the xy-plane is given by $(x(t), y(t)) = \left(t, \frac{1}{2}t^2\right)$.

      i. (2 pts) What is your elevation when $t = 2$?

      ii. (4 pts) Use a chain rule learned in this Calculus 3 class to determine whether your elevation is increasing or decreasing when $t = 2$.

4. [2350/062725 (6 pts)] Find the following limits or show that they do not exist.

   (a) $\displaystyle\lim_{(x,y,z)\to(\frac{\pi}{3},\frac{\pi}{6},\frac{\pi}{4})} \frac{\cos x + \sin y + \tan z}{\sqrt{3x + 6y + 4z}}$ (b) $\displaystyle\lim_{(x,y)\to(1,0)} \frac{1 - x}{x + y - 1}$

5. [2350/062725 (9 pts)] The dimensions of a closed rectangular box are measured as 3 ft, 4 ft, and 5 ft with a possible error of 1/100 ft in each. Use differentials to approximate the maximum error in the volume of the box.

MORE PROBLEMS BELOW/ON REVERSE

6. [2350/062725 (14 pts)] Jack and Jill went up a hill to the point (x_0, y_0, z_0) and got caught there in a lightning storm. In a fit of panic Jack darted off in the direction of $\mathbf{A} = 2\mathbf{i} - \mathbf{j}$ and noted at that instant that his altitude was changing at an instantaneous rate of $-\sqrt{5}/5$ ft/ft. Panic stricken as well, Jill ran in the direction of $\mathbf{B} = -3\mathbf{i} + \mathbf{j}$, noting an instantaneous rate of change of altitude of $-\sqrt{10}/5$ ft/ft. In what direction should they have run in order to start descending the hill at the fastest instantaneous rate? What would that rate have been?

7. [2350/062725 (10 pts)] Consider the function $g(x, y, z, t)$ where

$$x = u^2 + v, \quad y = u + v^2, \quad z = \ln(v/u), \quad t = e^{uv}$$

Suppose when $z = -\ln 4$ and $v = 1$ that $g_x = 2$, $g_y = -3$, $g_z = 6$ and $g_t = -2$. Calculate the instantaneous rate of change of $g(x, y, z, t)$ with respect to u at this point.

8. (16 pts) Consider the function $f(x, y) = \ln(xy)$.

   (a) (8 pts) Calculate the first order Taylor (tangent plane) approximation (linearization) of f about the point $(1, 1)$.

   (b) (2 pts) Approximate the value of $f(1.1, 1.2)$ using your first order Taylor polynomial.

   (c) (6 pts) Find an upper bound on the error in your first order Taylor approximation over the region where $|x - 1| \leq 0.2$ and $|y - 1| \leq 0.2$.