- This exam is worth 100 points and has 8 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/062725 (18 pts)] Consider the vectors $\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$ in $\mathbb{R}^3$.

   (a) (6 pts) Do the vectors form a basis for $\mathbb{R}^3$. Why or why not?

   (b) (6 pts) Find all solutions of $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$ where the columns of $\mathbf{A}$ are the vectors in the set in the order shown.

   (c) (6 pts) Based on your answer in (b), find the dimension of and a basis for the subspace of $\mathbb{R}^3$ consisting of all solutions of the equation $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$.

2. [2360/062725 (10 pts)] Consider the linear system

$$\begin{aligned} x_1 \qquad\quad + 3x_3 \qquad\;\; &= 1 \\ 2x_1 + \;\; x_2 + 4x_3 - x_4 &= 2 \\ 3x_1 + 2x_2 + 4x_3 \qquad\;\; &= -1 \\ 3x_2 - \;\; x_3 \qquad\;\; &= -2 \end{aligned}$$

   Use Cramer's Rule to find x_2. No points will be awarded for using any other technique.

3. [2360/062725 (10 pts)] Given that $\mathbb{M}_{22}$ is the vector space of all 2×2 matrices, determine if the following subsets, $\mathbb{W}$, are subspaces of $\mathbb{M}_{22}$. Justify your answers.

   (a) (5 pts) $\mathbb{W}$ is the set of matrices of the form $\begin{bmatrix} a & -b \\ b & c \end{bmatrix}$ where a, b, c are real numbers.

   (b) (5 pts) $\mathbb{W}$ is the set of matrices of the form $\begin{bmatrix} 2 & a \\ -a & 3 \end{bmatrix}$ where a is a real number.

4. [2360/062725 (18 pts)] If $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$, calculate the following, if possible. If not possible, simply write "not possible".

   Hint: for part (f), consider using properties of determinants.

   (a) $\mathbf{AB}$ (b) $\mathbf{B}^{\mathrm{T}}\mathbf{A}$ (c) $\mathbf{AA}^{-1}$ (d) $\left|\mathbf{A}^{\mathrm{T}}\mathbf{A}\right|$ (e) $(\mathbf{BA})^{\mathrm{T}}$ (f) $\left|\mathbf{BB}^{\mathrm{T}}\mathbf{B}^{-1}\right|$

MORE PROBLEMS BELOW/ON REVERSE

5. [2360/062725 (12 pts)] You are given the matrices $\mathbf{C}, \mathbf{D}$ and column vector $\vec{\mathbf{u}}$ as follows:

$$\mathbf{C} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & -1 & -1 \\ 2 & -1 & 0 \end{bmatrix} \qquad \vec{\mathbf{u}} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

(a) (5 pts) Compute $\mathbf{DC}$. Be sure to check your answer carefully.

(b) (7 pts) Without performing any elementary row operations or Gauss-Jordan Elimination, and applying what you found in part (a), find the solution of $\mathbf{C}\vec{\mathbf{x}} = \vec{\mathbf{u}}$. No points awarded if directions are not followed.

6. [2360/062725 (12 pts)] The augmented matrix of the linear system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ has been transformed, through a number of elementary row operations, to the following:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & k & 1 & 2 \\ 0 & 0 & k-1 & k^2-1 \end{array}\right]$$

where k is a real number parameter. For which value(s) of k, if any, does the system have ...

(a) (4 pts) exactly one solution?

(b) (4 pts) no solution?

(c) (4 pts) infinitely many solutions?

7. [2360/062725 (10 pts)] Determine if the set of vectors $\{1, 1-t, 2-4t+t^2, 6-18t+9t^2-t^3\}$ forms a basis for $\mathbb{P}_3$. Be sure to provide correct justification.

8. [2360/062725 (10 pts)] Let $\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -2 \\ 1 & 1 & 0 \end{bmatrix}$.

(a) (4 pts) Find all the eigenvalues of $\mathbf{A}$ and state the multiplicity of each.

(b) (6 pts) For the eigenvalue with algebraic multiplicity greater than 1, find its geometric multiplicity and a basis for its eigenspace.