# Python for Math and Stat Spring 2025
# Final Exam

Assume that all necessary packages have been imported.

1. (12 pts) For the following 4 problems, write down what each code block would display if executed in a Jupyter cell. If the code generates an error or infinite loop, write `Error`.

   (a)
   ```
   num_arr = np.array([1., 2., 3.])
   4*num_arr - num_arr**2 - 3
   ```

   (b)
   ```
   alpha = 'abcdefghijklmnopqrstuvwxyz'
   a = {alpha[x]: x+1 for x in range(len(alpha))}
   a['x']
   ```

   (c)
   ```
   list(map(lambda x: x**2 + 1, [-3, 0, 4]))
   ```

   (d)
   ```
   def func(num):
       print(num, end=' ')
       if num <= 0:
           return 3
       else:
           return 2*num + func(num-3)

   func(10)
   ```

   **Solution:**

   (a) `array([0., 1., 0.])`

   (b) `24`

   (c) `[10, 1, 17]`

   (d) `10 7 4 1 -2`
   `47`

2. (10 pts) The following two questions are related.

   (a) Write a function called **`count_threes(num)`** which returns the number of digits in positive integer `num` equal to 3. For example `count_threes(1138)` would return 1 while `count_threes(234353)` would return 3.

   (b) Write code to use your function to find the average (mean) number of 3s in all numbers from 1 to $10^7$.

**Solution:**

(a)
```
def count_threes(num):
    strnum = str(num)
    return strnum.count('3')


def count_threes(num):
    strnum = str(num)

    ct = 0
    for digit in strnum:
        if digit == '3':
            ct += 1

    return ct
```

(b)
```
sum3 = 0

for num in range(1, 10**7 + 1):
    sum3 += count_threes(num)

sum3 / 10**7
```

3. (12 pts) A *Pythagorean triple* $(x, y, z)$ of three positive integers has the property

$$x^2 + y^2 = z^2,$$

corresponding to the sides of a right triangle with hypotenuse $z$ and legs $x$ and $y$.

Euclid's method generates a Pythagorean triple by taking any two distinct positive integers $a$ and $b$ and producing these values for $x$ and $y$:

$$x = |a^2 - b^2|$$
$$y = 2ab$$

(a) Write a function called **`Pyth_trip(a, b)`** that takes two positive integers `a` and `b` and uses Euclid's method to generate a Pythagorean triple. The function returns the answer in the form of an `(x,y,z)` tuple of `int`s. You may assume `a` and `b` are distinct. For example, `Pyth_trip(1, 2)` returns `(3, 4, 5)`.

(b) A Pythagorean triple $(x, y, z)$ is *primitive* if $x$, $y$, and $z$ have no common divisor. Write a function **`Pyth_reduce(triple)`** that takes an `(x, y, z)` triple in the form of a tuple and returns the corresponding primitive triple. You may call the `gcd(m, n)` function from class to find the greatest common divisor of $x$ and $y$. For example, `Pyth_reduce((10, 24, 26))` returns `(5, 12, 13)`.

**Solution:**

(a)
```
def Pyth_triple(a, b):
     x = abs(a**2 - b**2)
     y = 2*a*b
     z = int(math.sqrt(x**2 + y**2))
     return x, y, z
```

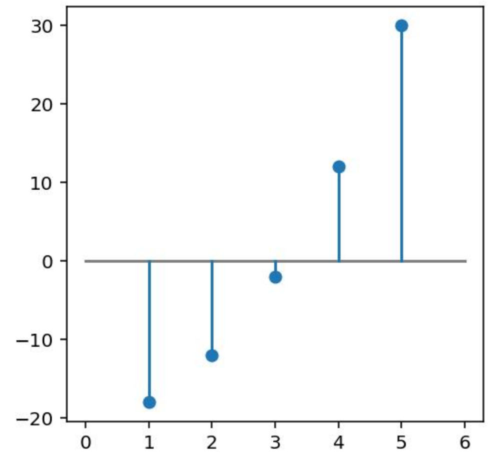
(b)
```
def Pyth_reduce(triple):
     (x, y, z) = triple
     divisor = gcd(x, y)

     return x // divisor, y // divisor, z // divisor
```

4. (14 pts)

(a) Write a function **`quad_pts(coeffs, npts)`** that takes the coefficients $(a, b, c)$ of a quadratic function $f(x) = ax^2 + bx + c$ and returns the function values `f(1), f(2), ..., f(npts)` in a numpy array. The code must use **numpy features** such as `arange` and vectorization. It should not use a loop or list comprehension.

Example: `quad_pts((2, 0, -20), 5)` returns `array([-18, -12, -2, 12, 30])` corresponding to $f(x) = 2x^2 - 20$, $f(1) = 18, \ldots, f(5) = 30$.

(b) Write a function **`quad_plot(coeffs, npts)`** that takes the output of `quad_pts`, plots the points, and draws a vertical line from each point to the $x$-axis. The code should plot a horizontal line to indicate where the $x$-axis is. Use default colors.




**Solution:**

(a)
```
def quad_pts(coeffs, npts):
    a, b, c = coeffs
    xvals = np.arange(1, npts+1)

    return a*xvals**2 + b*xvals + c
```

(b)
```
def quad_plot(coeffs, npts):
    xvals = np.arange(1, npts+1)
    yvals = quad_pts(coeffs, npts)

    # plot the x-axis
    plt.plot((0, npts+1), (0, 0))

    # plot the points
    plt.plot(xvals, yvals, 'o')

    # plot the vertical lines
    for x in xvals:
        plt.plot((x, x), (0, yvals[x-1]))
```

5. (15 pts) Create a class called **`Date`**. Each instance of the class corresponds to a date with month, day, and year attributes

   - **`month`**: an integer from 1 to 12
   - **`day`**: an integer from 1 to 31
   - **`year`**: a 4-digit integer

   Example: `vars(Date(5, 7, 2025))` returns `{'month': 5, 'day': 7, 'year': 2025}`.

   The class includes these methods:

   - **`month_name()`**: returns the name of the month in string format. The code can reference a global variable **`months`** = `['Jan', 'Feb', ..., 'Dec']` which contains the 3-letter abbreviations for month names in an ordered list.
     Example: `Date(5, 7, 2025).month_name()` returns `'May'`.
   - **`format()`**: returns the date in `m/d/yy` string format.
     Example: `Date(12, 4, 2025).format()` returns `'12/4/25'`.
   - **`century()`**: returns the century the given date is in. (For example, the 21st century includes the years 2001 to 2100 inclusive.)
     Examples:
     `Date(5, 7, 1776).century()` returns `18`, representing the 18th century.
     `Date(5, 7, 2000).century()` returns `20`, representing the 20th century.

**Solution:**

```
class Date:

    def __init__(self, month, day, year):
        self.month = month
        self.day = day
        self.year = year

    def month_name(self):
        return months[self.month - 1]

    def format(self):
        return f'{self.month:02}/{self.day:02}/{self.year % 100}'

    def century(self):
        hundred = self.year // 100
        if self.year % 100 != 0:
            hundred += 1
        return hundred
```

6. (12 pts) The DataFrame **`dfplanet`** contains information about the eight planets in our solar system. The DataFrame has an index column `Name` and columns `Large_Moons` for the number of large moons, `Icy_Moons` for the number of smaller icy moons, and `Rocks` for the number of asteroid-sized (very small) moons of each planet.

| | Large_Moons | Icy_Moons | Rocks |
|---|---|---|---|
| **Name** | | | |
| **Mercury** | 0 | 0 | 0 |
| | ... | | |
| **Uranus** | 0 | 5 | 20 |
| **Neptune** | 1 | 0 | 20 |

Write code to do the following:

(a) Determine the number of planets in `dfplanet` that have more `Large_Moons` than `Icy_Moons`.

(b) Select the names of all planets that have moons that are `Rocks`. The result should be a pandas index or a list of strings.

(c) Add a new column to the DataFrame called `Non_Rocks` which equals the number of `Large_Moons` plus `Icy_Moons`.

(d) Among the planets with `Large_Moons`, one has the largest number of `Rocks`. Identify the name of that planet as a string.

**Solution:**

(a)
```
len(dfplanet[dfplanet.Large_Moons > dfplanet.Icy_Moons])
```

OR
```
(dfplanet.Large_Moons > dfplanet.Icy_Moons).sum()
```

(b)
```
dfplanet[dfplanet.Rocks != 0].index
```

(c)
```
dfplanet['Non_Rocks'] = dfplanet.Large_Moons + dfplanet.Icy_Moons
```

(d)
```
dfplanet[dfplanet.Large_Moons > 0].Rocks.idxmax()
```