Answer the following problems, showing all of your work and simplifying your solutions where possible.

1. (48 pts) Evaluate the following integrals.

   (a) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\mathrm{d}x$

   (b) $\displaystyle\int_0^{\sqrt{2}} \frac{2x^2-4x+9}{(x+1)(x^2+2)}\mathrm{d}x$

   (c) $\displaystyle\int \sin(x)e^{2x}\mathrm{d}x$

   (d) $\displaystyle\int_0^{\pi/4} \sec^4(x)\tan^2(x)\mathrm{d}x$

2. (16 pts) Are the following integrals convergent or divergent? Justify your answers. If convergent, evaluate the integral if possible.

   (a) $\displaystyle\int_1^{\infty} \frac{|\cos(x)|}{x^{3/2}}\mathrm{d}x$

   (b) $\displaystyle\int_0^{3} \frac{1}{(x-1)^{4/3}}\mathrm{d}x$

3. (36 pts) Let $I = \displaystyle\int_0^{\pi} \cos^2\theta \mathrm{d}\theta$.

   (a) Estimate I with T_4, the trapezoidal rule with $n=4$.

   (b) Find a reasonable bound for $|E_T|$ for your calculation in part (a).

   (c) What is the minimum number of intervals required so the error of T_n is less than $\frac{1}{100}$?

   (d) Compute the exact value of I. What is the true error of your estimate in part (a)?

   (e) Is there is a large difference between your bound for $|E_T|$ and the true error? Why might this occur?

Trigonometric Identities

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$ $\sin^2(x) = \frac{1}{2}(1-\cos(2x))$ $\sin(2x) = 2\sin(x)\cos(x)$ $\cos(2x) = \cos^2(x) - \sin^2(x)$

Inverse Trigonometric Integral Identities

$$\int \frac{\mathrm{d}u}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C,\ u^2 < a^2$$

$$\int \frac{\mathrm{d}u}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{\mathrm{d}u}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C,\ u^2 > a^2$$

Midpoint Rule

$$\int_a^b f(x)\mathrm{d}x \approx \Delta x[f(\overline{x_1}) + f(\overline{x_2}) + ... + f(\overline{x_n})],\ \Delta x = \frac{b-a}{n}, \overline{x_i} = \frac{x_{i-1}+x_i}{2},\ |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Trapezoidal Rule

$$\int_a^b f(x)\mathrm{d}x \approx \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+...+2f(x_{n-1})+f(x_n)],\ \Delta x = \frac{b-a}{n},\ |E_T| \leq \frac{K(b-a)^3}{12n^2}$$