- This exam is worth 100 points and has 7 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/061325 (12 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) $(\mathbf{u}\cdot\mathbf{v})\times\mathbf{w} = \mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})$

   (b) The cross product of two nonzero vectors that are scalar multiples of each other has magnitude 0.

   (c) $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$

   (d) The intersection of the plane $z = 2$ and the surface $4x^2 + y^2 + 4z^2 - 4y - 24z + 36 = 0$ is an ellipse.

   (e) The normal component of the acceleration of a particle moving along a straight line is always zero.

   (f) The unit binormal vector $\mathbf{B}$ for a curve lying in the plane $z = 3$ is $\pm\mathbf{k}$.

2. [2350/061325 (16 pts)] A particle travels along the helix given by $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k}$. At time $t = \pi$ the particle leaves the path and flies off on a tangent. Find the location of the particle at $t = 2\pi$ assuming no forces act on it after it leaves the helix.

3. [2350/061325 (10 pts)] Find the equation of, and identify, the quadric surface whose points are equidistant from the point $P_0(2,0,0)$ and the plane containing the point $(-2,0,0)$ whose normal vector is $\mathbf{i}$.

4. [2350/061325 (20 pts)] Consider the vector function $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$ with $-\infty < t < \infty$.

   (a) (10 pts) Compute the *torsion*, τ (the measure of the degree of twisting of a curve), given by $\tau = \dfrac{(\mathbf{r}'\times\mathbf{r}'')\cdot\mathbf{r}'''}{\|\mathbf{r}'\times\mathbf{r}''\|^2}$ at the point $(2,4,8)$.

   (b) (10 pts) Are there any points on the curve where the velocity and acceleration vectors are orthogonal? If so, find them. If not, explain why not.

5. [2350/061325 (10 pts)] Find the position vector $\mathbf{r}(t)$ of an object subject to the following conditions: it undergoes an acceleration of $e^t\,\mathbf{i} + 2t\,\mathbf{j} + (t+1)\,\mathbf{k}$ for $t \geq 0$ and it begins its motion at $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ with a velocity of $\mathbf{i} + \mathbf{k}$.

6. [2350/061325 (24 pts)] Consider the intersecting lines $L_1(t) = \langle 7-2t, t, -4-t\rangle$ and $L_2(s) = \langle 3+2s, -3+4s, -8+3s\rangle$.

   (a) (6 pts) Find the coordinates of the point where the lines intersect.

   (b) (6 pts) Find the equation of the plane containing the lines. Write your final answer in the form $az + by + cz = d$.

   (c) (6 pts) Find the symmetric equations of the line normal to the plane you found in part (b) and passing through the point you found in part (a).

   (d) (6 pts) Find the coordinates of the point where the line from part (c) intersects the plane $x + y + z = 2$.

7. [2350/061325 (8 pts)] Consider the vector function $\mathbf{r}(t) = \langle t^2, \sin t - t\cos t, \cos t + t\sin t\rangle$ for $0 \leq t \leq c$. Find the value of c such that the arc length at $t = c$ is $8\sqrt{5}$.