APPM 3570/STAT 3100 **Exam 3 Section 002** **Spring 2025**

NAME: _____

SECTION: (Circle One) 001 at 10:10 am *or* 002 at 2:30 pm

Instructions:

1. Calculators are permitted.
2. Notes, your text and other books, cell phones, and other electronic devices are not permitted—except for calculators or as needed to view and upload your work.
3. Justify your answers, show all work.
4. When you have completed the exam, go to the uploading area in the room and scan your exam and upload it to Gradescope.
5. Don't forget to scan any pages you used for extra space!
6. Verify that everything has been uploaded correctly and the pages have been associated to the correct problems.
7. Turn in your hardcopy exam.

On my honor as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work.

Signature: _____ Date: _____

Duration: 90 minutes

Problem 1. (28 points.) Consider a model in which the joint pdf of city populations x (in millions) obeys a distribution with parameter y, which itself is randomly drawn. The joint density function for X and Y is:

$$f_{X,Y}(x,y) = \begin{cases} y(y+1)\frac{e^{-y}}{x^{y+2}} & 1 \leq x < \infty,\ 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the marginal density of Y?

(b) What is the conditional density of X given that $Y = y$?

(c) Determine the conditional expectation $\mathrm{E}[X|Y]$.

(d) Determine the total expectation $\mathrm{E}[X]$.

(e) Are X and Y independent? Justify your answer.

(Use the back if additional space is needed!)

Problem 2. (15 points.) An organism has a lifetime X (in days) distributed exponentially $X \sim \textit{Exponential}(1)$, so $f_X(x) = e^{-x}$ for $x > 0$.

The organism can have a single offspring at time Y (in days) also distributed exponentially $Y \sim \textit{Exponential}(2)$, so $f_Y(y) = 2e^{-2y}$ for $y > 0$.

Notice that if $X < Y$, the organism will have no offspring, and its lineage terminates.

Assume that X and Y are independent.

Determine the probability an organism dies before it has an offspring, $\mathrm{P}(X < Y)$.

(Use the back if additional space is needed!)

Problem 3. (22 points.) Suppose a barn has 10 stalls, all in a row. Suppose 6 horses and 4 goats are each led into a stall, randomly and independently, and each stall holds exactly one animal.

What is the expected number of horses that are in a stall next to a goat?

(Use the back if additional space is needed!)

Problem 4. (25 points.) A bank teller serves customers standing in a queue one by one. Suppose that the service time in minutes for a customer follows an exponential distribution with $\lambda = \frac{1}{2}$ as the rate parameter. Assume that service times for different bank customers are independent.

(a) Describe the approximate distribution of the total time that the bank teller will spend serving 50 randomly selected customers. Justify any assumptions.

(b) Estimate the probability that the total time that the bank teller will spend serving 50 randomly selected customers is between 90 and 95 minutes.

(Use the back if additional space is needed!)

Problem 5. (10 points.) Let $X = V + W$ and $Y = V + Z$, where V, W, and Z are independent and identically distributed (i.i.d.) Poisson random variables with parameter λ.

(a) Find $\mathrm{E}[X + Y]$.

(b) Find $\mathrm{Cov}(X, Y)$.

(c) Are X and Y independent? Justify your answer.

(Use the back if additional space is needed!)

Standard Normal Table

$\Phi(z) = P(Z \leq z)$ for $Z \sim N(0, 1)$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990