

1. (20 pts) The following parts are unrelated.

- (a) Find $\frac{dy}{dx}$ for $y = \ln(7 - 3x^4)$.
- (b) Find y' for: $xe^y - \cosh(y) = 399$. You may leave hyperbolic function(s) in your answer.

2. (36 pts) The following are unrelated.

- (a) Evaluate the limit: $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^{3x}$
- (b) Evaluate the limit: $\lim_{x \rightarrow 0} \frac{\arcsin(x)}{x}$
- (c) Evaluate the definite integral $\int_1^{\ln(2)} \frac{5e^x}{e^x + 1} dx$
- (d) Evaluate the indefinite integral $\int \frac{\sin(\theta)}{1 + \cos^2(\theta)} d\theta$

3. (16 pts) Consider the function $f(x) = 5x^2 + 2x - 3$ on the interval $[-1, 2]$.

- (a) Approximate $\int_{-1}^2 f(x) dx$ using two rectangles of equal width with the right end point rule (R_2).
- (b) State the hypothesis of the Mean Value Theorem and explain why $f(x)$ satisfies those hypotheses.
- (c) Find all numbers, c , that satisfy the conclusion of the Mean Value Theorem.

4. (16 pts) For $f(x) = \int_{2x}^1 \sin^{-1}(t) dt$ answer the following:

- (a) Find $f'(x)$.
- (b) Find the equation of the line tangent to f that passes through the point $\left(\frac{1}{4}, \frac{5\pi - 6\sqrt{3}}{12}\right)$.
- (c) Find $f''(x)$.

5. (18 pts) The position, s measured in feet, of a particle moving in a straight line is given by $s(t) = \frac{t^3}{3} - \frac{t^2}{2} - 6t$ for time t in seconds. The particle starts moving at $t = 0$ and the equation is valid for $t \geq 0$. Answer the following and be sure to include units below where relevant.

- (a) Find the velocity of the particle as a function of t .
- (b) On what interval(s) of time is the particle moving in the positive direction?
- (c) Find the acceleration of the particle at 3 seconds.
- (d) Find the average velocity of the particle on the interval $[0, 3]$.
- (e) Find the total distance traveled by the particle during the first 4 seconds.

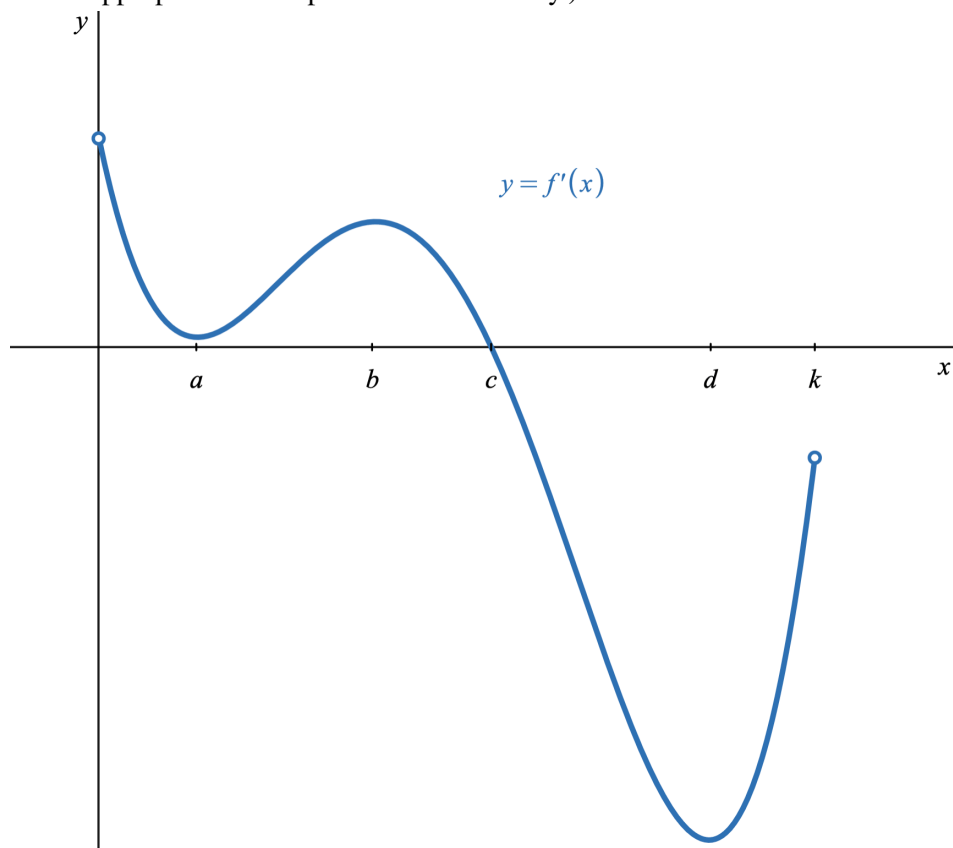
6. (24 pts) The following parts are unrelated.

- (a) A bacteria culture initially contains 106 cells and its population, $P(t)$, grows at a rate proportional to its size. After an hour the population has increased to 420. Find a function for the number of cells after t hours.

- (b) At noon, ship A is 60 km west of ship B. Ship A is sailing south at 15 km/h and ship B is sailing north at 5 km/h. How fast is the distance between the ships changing at 4:00 PM?

7. (20 pts) Parts (a) and (b) are unrelated. Part (b) is on the next page.

- (a) Below is the graph of the **first derivative**, f' , of a function f . Answer the following questions related to f which is defined on $(0, k)$ with x -values: a , b , c , d , and k . (List all answers that apply. Use interval notation where appropriate. No explanation is necessary.)



- Evaluate $\lim_{h \rightarrow 0} \frac{f'(b+h) - f'(b)}{h}$
- On what intervals is f decreasing?
- At what x -value does f have a local maximum?
- On what intervals is f concave down?
- At what value(s) of x does f have an inflection point?

(b) Sketch a graph of a **single function** $y = g(x)$ with **all** of the following properties:

- $g(0) = 1$
- $\lim_{x \rightarrow 1^-} g(x) = -\infty$
- $\lim_{x \rightarrow \infty} g(x) = 2$
- $g'(x) > 0$ if $x < 0$ and $x \neq -1, -2$
- $g(-x) = g(x)$
- $\lim_{x \rightarrow -1^-} g(x) = +\infty$
- $\lim_{x \rightarrow -2} g(x) = 3$
- $g(-2)$ DNE