1. Simplify each of the following. Leave answers without negative exponents. (13 pts)

   (a) $x(x+1)-(-2x-1)(x+1)$
   Solution:

$$\begin{aligned} x(x+1)-(-2x-1)(x+1) &= x(x+1)+(2x+1)(x+1) && (1)\\ &= (x+1)(x+2x+1) && (2)\\ &= (x+1)(3x+1) && (3)\\ &= \boxed{3x^2+4x+1} && (4) \end{aligned}$$

   (b) $\sqrt{18x^4y}$
   Solution:

$$\begin{aligned} \sqrt{18x^4y} &= \sqrt{3.3.2.x^2.x^2.y} && (5)\\ &= \boxed{3x^2\sqrt{2y}} && (6) \end{aligned}$$

   (c) $\dfrac{3-\frac{1}{2^2}+1}{4+\frac{1}{2^0}}$
   Solution:

$$\begin{aligned} \frac{3-\frac{1}{2^2}+1}{4+\frac{1}{2^0}} &= \frac{4-\frac{1}{4}}{4+\frac{1}{1}} && (7)\\ &= \frac{\frac{15}{4}}{5} && (8)\\ &= \boxed{\frac{3}{4}} && (9) \end{aligned}$$

   (d) $(e^x+y)^2-2ye^x-\ln\left(e^{y^2}\right)$
   Solution:

$$\begin{aligned} (e^x+y)^2-2ye^x-\ln\left(e^{y^2}\right) &= e^{2x}+2e^xy+y^2-2e^xy-y^2 && (10)\\ &= \boxed{e^{2x}} && (11) \end{aligned}$$

2. Simplify each of the following. Leave answers without negative exponents. (8 pts)

   (a) $\left(\dfrac{16a^{-2}b^3}{2^{-1}a^0b^{-4}}\right)(ab)^{-2}$
   Solution:

$$\begin{aligned} \left(\frac{16a^{-2}b^3}{2^{-1}a^0b^{-4}}\right)(ab)^{-2} &= \left(\frac{16b^3b^42^1}{a^2}\right)\frac{1}{a^2b^2} && (12)\\ &= \boxed{\frac{32b^5}{a^4}} && (13) \end{aligned}$$

   (b) $\left(m^{2/3}+n^{2/3}\right)\left(m^{4/3}-m^{2/3}n^{2/3}+n^{4/3}\right)$
   Solution:

$$\begin{aligned} \left(m^{2/3}+n^{2/3}\right)\left(m^{4/3}-m^{2/3}n^{2/3}+n^{4/3}\right) &= \left(m^{2/3}+n^{2/3}\right)\left((m^{2/3})^2-m^{2/3}n^{2/3}+(n^{2/3})^2\right) && (14)\\ &= \left(m^{\frac{2}{3}}\right)^3+\left(n^{\frac{2}{3}}\right)^3 && (15)\\ &= \boxed{m^2+n^2} && (16) \end{aligned}$$

3. Solve $(t-1)^2 = 8$ for t. (5 pts)
Solution:

$$(t-1)^2 = 8 \tag{17}$$
$$t-1 = \pm\sqrt{8} \tag{18}$$
$$t = 1 \pm \sqrt{8} \tag{19}$$
$$t = \boxed{1 \pm 2\sqrt{2}} \tag{20}$$

4. Solve the following equations for x: (15 pts)

(a) $1 = x + \sqrt{7-x}$ **Solution:**

$$1 = x + \sqrt{7-x} \tag{21}$$
$$1 - x = \sqrt{7-x} \tag{22}$$
$$(1-x)^2 = 7 - x \tag{23}$$
$$1 - 2x + x^2 = 7 - x \tag{24}$$
$$x^2 - x - 6 = 0 \tag{25}$$
$$(x-3)(x+2) = 0 \tag{26}$$

This results in potential answers $x = -2, 3$. Checking these values in the original equation, we see that only $x = -2$ solves the original equation. Hence the solution is $\boxed{x = -2}$

(b) $\dfrac{1}{x-1} - \dfrac{\sqrt{2}}{x+1} = \dfrac{1}{x^2-1}$

Solution:

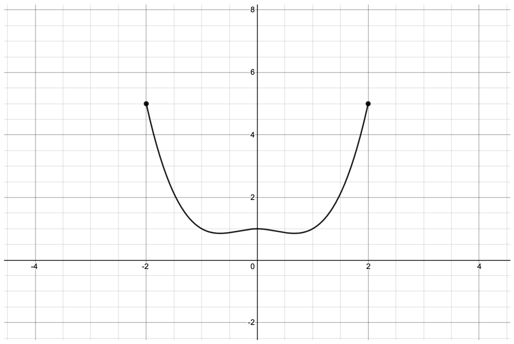
$$\frac{1}{x-1} - \frac{\sqrt{2}}{x+1} = \frac{1}{x^2-1} \tag{27}$$
$$(x+1)(x-1)\left(\frac{1}{x-1} - \frac{\sqrt{2}}{x+1}\right) = (x+1)(x-1)\left(\frac{1}{x^2-1}\right) \tag{28}$$
$$(x+1) - \sqrt{2}(x-1) = (x+1)(x-1)\frac{1}{(x+1)(x-1)} \tag{29}$$
$$x\left(1-\sqrt{2}\right) + \sqrt{2} + 1 = 1 \tag{30}$$
$$x\left(1-\sqrt{2}\right) + \sqrt{2} = 0 \tag{31}$$
$$x = \boxed{\frac{\sqrt{2}}{\sqrt{2}-1}} \tag{32}$$

(c) $\log(3) - \log(2) = \log(2x+1)$
Solution:

$$\log(3) - \log(2) = \log(2x+1) \tag{33}$$
$$\log\frac{3}{2} = \log(2x+1) \tag{34}$$
$$\frac{3}{2} = 2x + 1 \tag{35}$$
$$2x = \frac{3}{2} - 1 \tag{36}$$
$$x = \boxed{\frac{1}{4}} \tag{37}$$

5. Parts (a), (b), and (c) below are unrelated from each other. (14 pts)

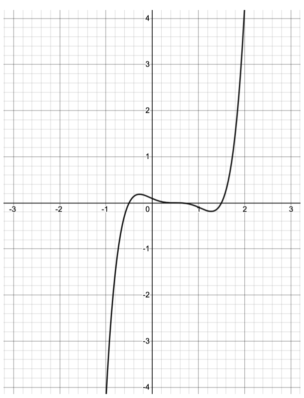
(a) i. For the graph of the function $g(x)$ below, with domain $[-2, 2]$, is this the graph of an odd, even, or neither function? No justification is needed.



Solution:
$\boxed{\text{Even}}$

ii. Is $g(x)$ one-to-one? As usual, justify your answer for credit.
Solution:
$\boxed{g(x) \text{ is not one-to-one because it fails the horizontal line test. Hence it does not have an inverse.}}$

(b) For the graph of $h(x)$ below is this the graph of an odd, even, or neither function? No justification is needed.



Solution:
$\boxed{\text{Neither}}$

(c) Is $k(x) = \sin x + x^3 - x$ symmetric about the y-axis, the origin, or neither? As usual justify your answer for credit.
Solution:

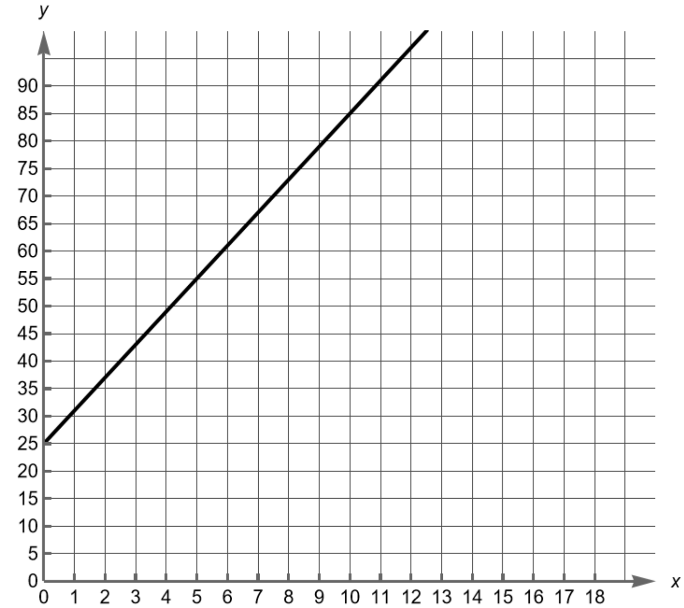
$$\begin{aligned} k(-x) &= \sin(-x) + (-x)^3 - (-x) && (38) \\ &= -\sin x - x^3 + x && (39) \\ &= -(\sin x + x^3 - x) && (40) \\ &= -k(x) && (41) \end{aligned}$$

Hence $k(x)$ is an odd function, and $\boxed{\text{symmetric about the origin.}}$

6. A toy manufacturer finds that if she produces x toys in a month, her production cost, C, is given by the linear equation $C = 6x + 25$. (8 pts)

(a) Sketch a graph of the linear equation.

Solution:



(b) What does the slope of the graph represent?

Solution:

The slope of the graph represents the rate of change of production cost w.r.t the number of toys produced.

In other words, it represents how much the production cost increases for each extra toy produced.

(c) What does the C-intercept of the graph represent?

Solution:

The C-intercept of the graph represents the the production cost when no toy is produced. It is called the fixed cost..

7. Consider the function $P(x) = -x^4 + 3x^3 + 4x^2$. Answer the following: (12 pts)

(a) Find all x and y-intercept(s).
Solution:
The x-intercept is obtained by solving

$$y = 0 \tag{42}$$
$$-x^4 + 3x^3 + 4x^2 = 0 \tag{43}$$
$$-x^2(x^2 - 3x - 4) = 0 \tag{44}$$
$$-x^2(x-4)(x+1) = 0 \tag{45}$$

Hence the zeros of the polynomial are:
$x = 0$ with a multiplicity of 2
$x = -1$ with a multiplicity of 1
$x = 4$ with a multiplicity of 1

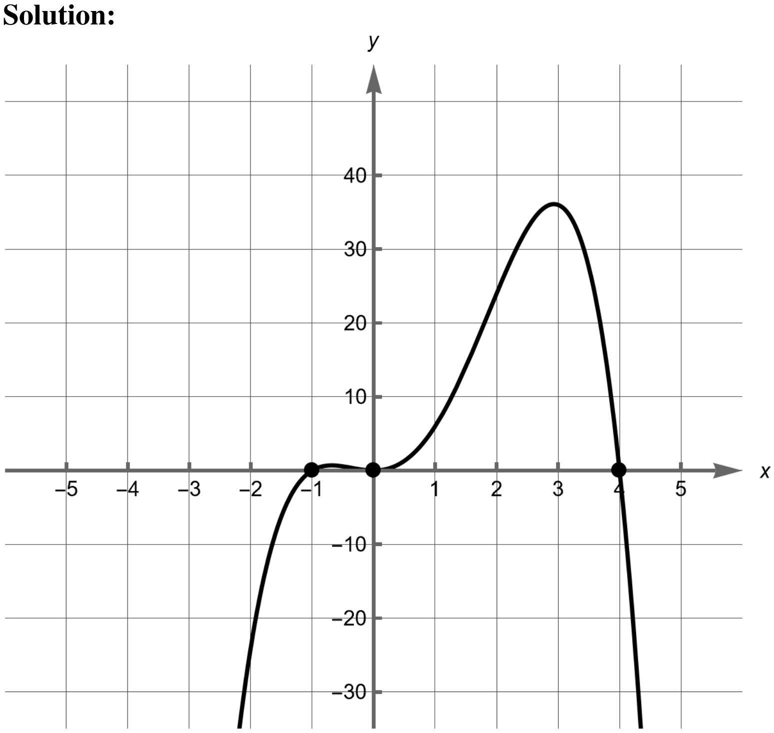
Hence the x-intercepts are $\boxed{(0,0),\ (-1,0), \text{and } (4,0)}$

The y-intercept is found by setting x to 0. We notice that the y-intercept is $\boxed{(0,0)}$

(b) Determine whether the graph bounces or crosses at each x-intercept.
Solution:
From the multiplicities of the xeroes listed above, we see that the graph $\boxed{\text{bounces at } (0,0)}$ (even multiplicity) and $\boxed{\text{crosses at } (-1,0) \text{ and } (4,0)}$ (odd multiplicity)

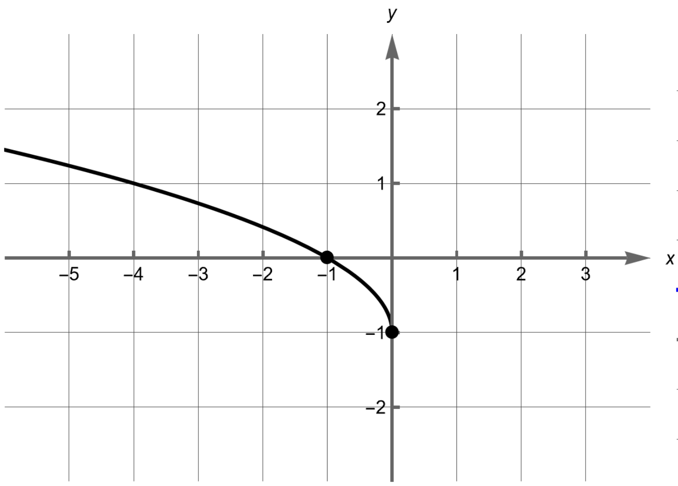
(c) Identify the end behavior (either using arrow notation or depicting on a graph).
Solution:
The end behavior is controlled by the leading term: $-x^4$. So $P(x) \to -\infty$ as $x \to -\infty$ and $P(x) \to -\infty$ as $x \to \infty$.

(d) Sketch the graph of $P(x)$ be sure to label all x and y-intercepts.
Solution:

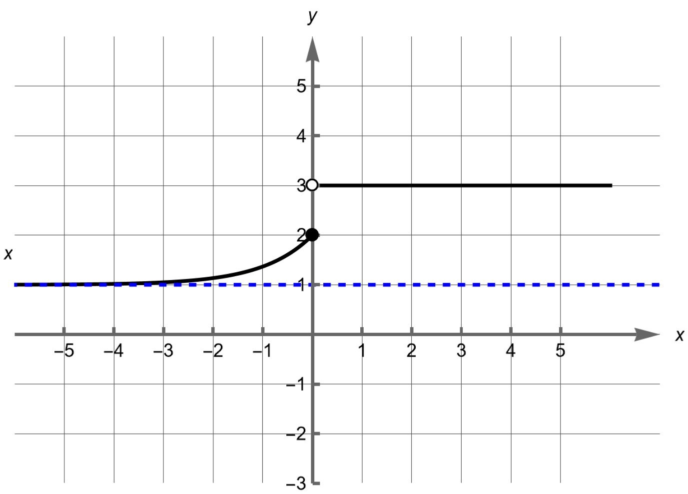


8. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (13 pts)

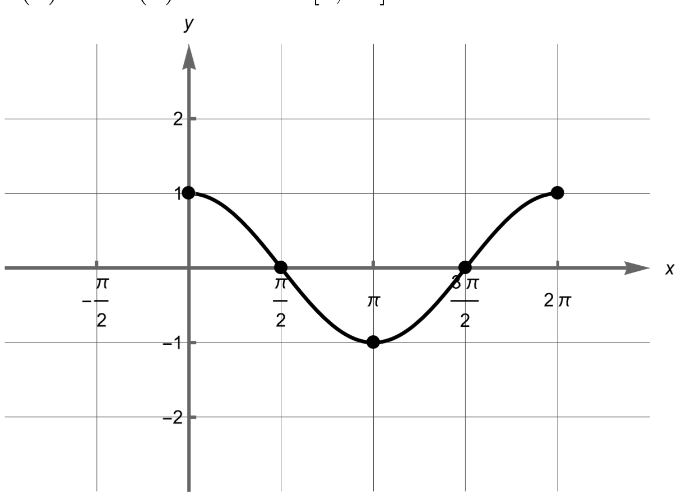
(a) $f(x) = \sqrt{-x} - 1$.



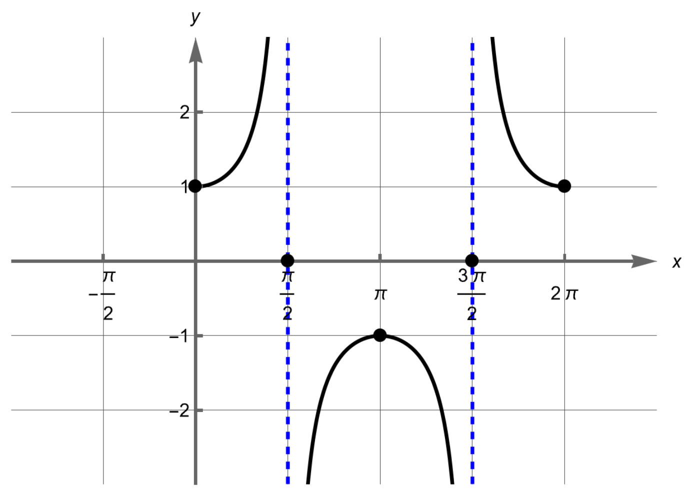
(b) $q(x) = \begin{cases} e^x + 1 & \text{if} \quad x \leq 0 \\ 3 & \text{if} \quad x > 0 \end{cases}$



(c) $h(x) = \cos(x)$ on domain $[0, 2\pi]$



(d) $k(x) = \sec(x)$ for one cycle



9. Given $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x-5}$ answer the following. (8 pts)

(a) Find $(f+g)(6)$.
Solution:

$$
\begin{aligned}
(f+g)(6) &= f(6) + g(6) && (46)\\
&= \frac{1}{6} + \frac{1}{1} && (47)\\
&= \boxed{\frac{7}{6}} && (48)
\end{aligned}
$$

(b) Find $(f \circ g)(x)$ and find the domain.
Solution:

$$
\begin{aligned}
(f \circ g)(x) &= f\left(g(x)\right) && (49)\\
&= \frac{1}{\frac{1}{x-5}} && (50)\\
&= \boxed{x-5} && (51)
\end{aligned}
$$

Since we are not allowed to divide by 0, the operations we performed require that $x \neq 5$. Hence the domain is $\boxed{(-\infty, 5) \cup (5, \infty)}$

(c) Find $(g \circ f)(x)$ and find the domain.
Solution:

$$(g \circ f)(x) = g(f(x)) \tag{52}$$
$$= \frac{1}{\left(\frac{1}{x} - 5\right)} \tag{53}$$
$$= \frac{1}{\left(\frac{1-5x}{x}\right)} \tag{54}$$
$$= \boxed{\frac{x}{1-5x}} \tag{55}$$

Since we are not allowed to divide by 0, the operations we performed require that $x \neq 0$ and $1 - 5x \neq 0$, i.e, $x \neq \frac{1}{5}$. Hence the domain is $\boxed{(-\infty, 0) \cup \left(0, \frac{1}{5}\right) \cup \left(\frac{1}{5}, \infty\right)}$

10. Find the exact value: (18 pts)

(a) $\cos\left(\frac{11\pi}{6}\right)$
Solution:

$$\boxed{\cos\left(\frac{11\pi}{6}\right) = \frac{\sqrt{3}}{2}}$$

(b) $\sin\left(\frac{3\pi}{2}\right)$
Solution:

$$\boxed{\sin\left(\frac{3\pi}{2}\right) = -1}$$

(c) $\sec\left(-\frac{4\pi}{3}\right)$
Solution:

$$\boxed{\sec\left(-\frac{4\pi}{3}\right) = -2}$$

(d) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$
Solution:

$$\boxed{\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{2}}$$

(e) $\tan^{-1}(-1)$

Solution:

$$\boxed{\tan^{-1}(-1) = -\frac{\pi}{4}}$$

(f) $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

Solution:

$$\boxed{\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) = \frac{\pi}{4}}$$

11. Verify the identity: $\sin^2\theta \tan\theta = \tan\theta - \sin\theta\cos\theta$ (5 pts)

(Hint: Start with the right hand side and recall the definition of $\tan\theta$)
Solution:
Starting with the Right Hand side

$$\tan\theta - \sin\theta\cos\theta = \frac{\sin\theta}{\cos\theta} - \sin\theta\cos\theta \tag{56}$$
$$= \sin\theta\left(\frac{1}{\cos\theta} - \cos\theta\right) \tag{57}$$
$$= \sin\theta\left(\frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta}\right) \tag{58}$$
$$= \sin\theta\left(\frac{1-\cos^2\theta}{\cos\theta}\right) \tag{59}$$
$$= \frac{\sin\theta\sin^2\theta}{\cos\theta} \tag{60}$$
$$= \boxed{\sin^2\theta\tan\theta} \tag{61}$$

12. Find all solutions to the following equations: (10 pts)

   (a) $\sqrt{3}\cos\theta - 2\sin\theta\cos\theta = 0$
   Solution:

$$\sqrt{3}\cos\theta - 2\sin\theta\cos\theta = 0 \tag{62}$$
$$\cos\theta\left(\sqrt{3} - 2\sin\theta\right) = 0 \tag{63}$$

   By multiplicative property of zero we get two equations (letting k as any integer)

   i. $\cos\theta = 0$
   This has solutions $\theta = \dfrac{\pi}{2} + 2k\pi$ and $\theta = \dfrac{3\pi}{2} + 2k\pi$. These can be combined into $\boxed{\theta = \dfrac{\pi}{2} + k\pi}$

   ii.

$$\sqrt{3} - 2\sin\theta = 0 \tag{64}$$
$$\sin\theta = \frac{\sqrt{3}}{2} \tag{65}$$

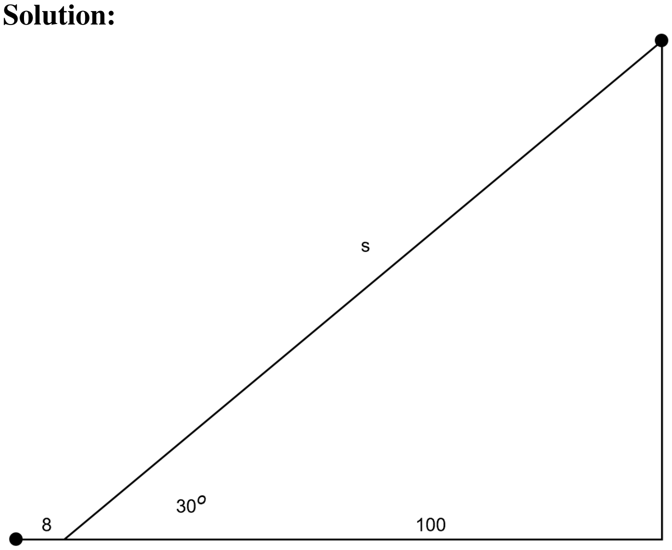
   This has solutions $\boxed{\theta = \dfrac{\pi}{3} + 2k\pi}$ and $\boxed{\theta = \dfrac{2\pi}{3} + 2k\pi}$

   (b) $\sin\left(\dfrac{\theta}{3}\right) = \dfrac{1}{2}$
   Solution:
   Using the unit circle, we obtain the following solutions (where k is any integer)

$$\frac{\theta}{3} = \frac{\pi}{6} + 2k\pi \tag{66}$$
$$\theta = \boxed{\frac{\pi}{2} + 6k\pi} \tag{67}$$

13. A bird, perched on a sheer cliff, spots a beetle on the flat ground 100 feet away from the base of the cliff. The bird flies straight to the beetle, snatches it up in its beak, and then runs 8 feet along the ground to join its flock. Suppose 30° is the angle between the bird's flight path and the ground. From the time the bird took flight, how far did the bird travel when it joined its flock? (5 pts)

   Solution:



From the right triangle above, we see that

$$\cos(30^o) = \frac{100}{s} \tag{68}$$
$$\frac{\sqrt{3}}{2} = \frac{100}{s} \tag{69}$$
$$s = \frac{200}{\sqrt{3}} \tag{70}$$

Hence the distance traveled by the bird along its flight path is $\frac{200}{\sqrt{3}}$ ft. The total distance traveled is $\boxed{\left(\frac{200}{\sqrt{3}} + 8\right) \text{ ft}}$

14. Find the exact value for each: (8 pts)

(a) $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$
Solution:
By a double angle formula

$$\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right) = \cos\left(2\frac{\pi}{8}\right) \tag{71}$$
$$= \cos\left(\frac{\pi}{4}\right) \tag{72}$$
$$= \boxed{\frac{\sqrt{2}}{2}} \tag{73}$$

(b) $\cos\left(-\frac{\pi}{12}\right)$
Solution:
Since cosine is an even function, $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{12}\right)$.
Then we notice that $\cos\left(\frac{\pi}{12}\right)$ is positive because $\frac{\pi}{12}$ is in the 1st quadrant. By a half angle formula, we can then write

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\frac{\pi}{6}}{2}\right) \tag{74}$$
$$= \sqrt{\frac{1+\cos\left(\frac{\pi}{6}\right)}{2}} \tag{75}$$
$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} \tag{76}$$
$$= \sqrt{\frac{2+\sqrt{3}}{4}} \tag{77}$$
$$= \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}} \tag{78}$$

Alternative Solution:

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4})\right) \tag{79}$$
$$= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \tag{80}$$
$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} \tag{81}$$
$$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}} \tag{82}$$

15. For $f(x) = 3\sin\left(x - \frac{\pi}{3}\right)$ (8 pts)

(a) Identify the amplitude.
Solution:
$\boxed{3}$

(b) Identify the period.
Solution:
$\boxed{2\pi}$

(c) Identify the phase shift.
Solution:
$\boxed{\frac{\pi}{3}}$

(d) Sketch one cycle of the graph of $f(x)$. Label the phase shift and ending x-value of the cycle on the x-axis and amplitude values on the y-axis to receive full credit.
Solution: