INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **justify all answers**. A correct answer with incorrect work or no justification may receive no credit. Books, notes, electronic devices, other unauthorized devices, and help from another person are not permitted while taking the exam. The final exam is worth 150 points.

**NOTE:** YOU MAY TEAR OFF THIS FIRST PAGE AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH FOLLOWING EXAM PAGE

iii. WRITE YOUR NAME ON THE NEXT PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND FOLLOW PROCTOR INSTRUCTIONS IN UPLOADING YOUR EXAM WITH SUPPORTING WORK TO GRADESCOPE. ONLY WORK THAT'S SUBMITTED TO GRADESCOPE WILL BE GRADED.

Formulas that may be useful:

1. $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

Name: _____

1. Simplify each of the following. Leave answers without negative exponents. (13 pts)

   (a) $x(x+1)-(-2x-1)(x+1)$

   (b) $\sqrt{18x^4y}$

   (c) $\dfrac{3-\frac{1}{2^2}+1}{4+\frac{1}{2^0}}$

   (d) $(e^x+y)^2-2ye^x-\ln\left(e^{y^2}\right)$

2. Simplify each of the following. Leave answers without negative exponents. (8 pts)

(a) $\left(\dfrac{16a^{-2}b^{3}}{2^{-1}a^{0}b^{-4}}\right)(ab)^{-2}$

(b) $\left(m^{2/3}+n^{2/3}\right)\left(m^{4/3}-m^{2/3}n^{2/3}+n^{4/3}\right)$

3. Solve $(t-1)^2 = 8$ for $t$. (5 pts)

4. Solve the following equations for $x$: (15 pts)

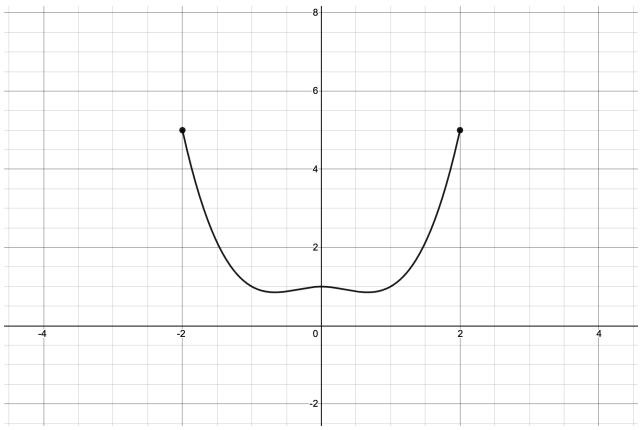
(a) $1 = x + \sqrt{7 - x}$

(b) $\dfrac{1}{x-1} - \dfrac{\sqrt{2}}{x+1} = \dfrac{1}{x^2-1}$

(c) $\log(3) - \log(2) = \log(2x + 1)$

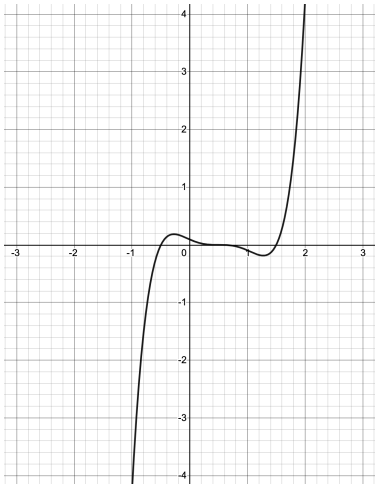
5. Parts (a), (b), and (c) below are unrelated from each other. (14 pts)

(a) i. For the graph of the function $g(x)$ below, with domain $[-2, 2]$, is this the graph of an odd, even, or neither function? No justification is needed.




ii. Is $g(x)$ one-to-one? As usual, justify your answer for credit.

(b) For the graph of $h(x)$ below is this the graph of an odd, even, or neither function? No justification is needed.




(c) Is $k(x) = \sin x + x^3 - x$ symmetric about the $y$-axis, the origin, or neither? As usual justify your answer for credit.

6. A toy manufacturer finds that if she produces $x$ toys in a month, her production cost, $C$, is given by the linear equation $C = 6x + 25$. (8 pts)

   (a) Sketch a graph of the linear equation.

   (b) What does the slope of the graph represent?

   (c) What does the $C$-intercept of the graph represent?

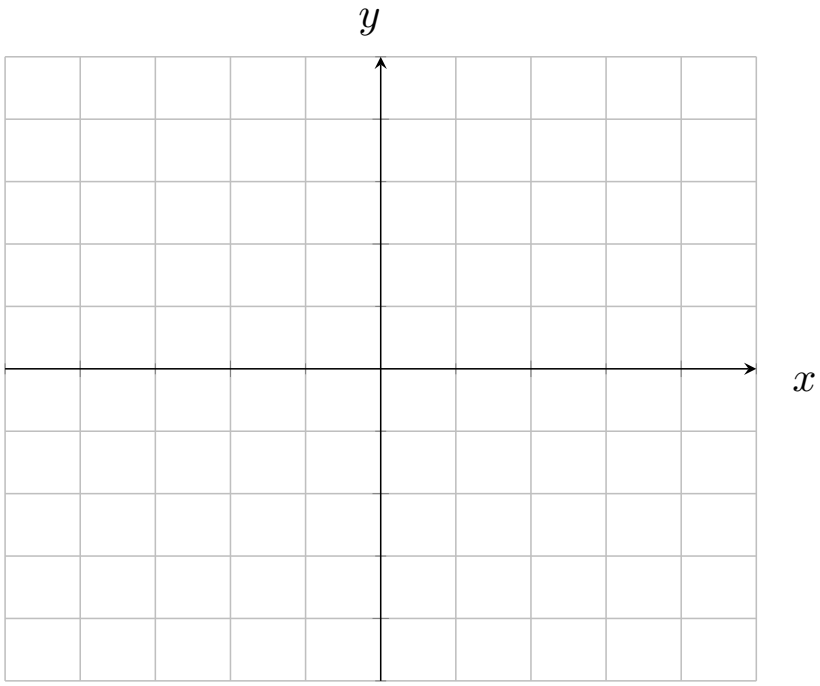
7. Consider the function $P(x) = -x^4 + 3x^3 + 4x^2$. Answer the following: (12 pts)

(a) Find all $x$ and $y$-intercept(s).

(b) Determine whether the graph bounces or crosses at each $x$-intercept.

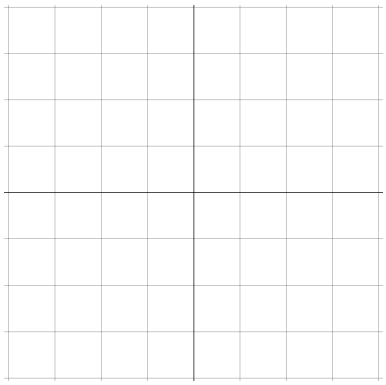
(c) Identify the end behavior (either using arrow notation or depicting on a graph).

(d) Use parts (a)-(c) to sketch the graph of $P(x)$ and be sure to label all $x$ and $y$-intercepts.

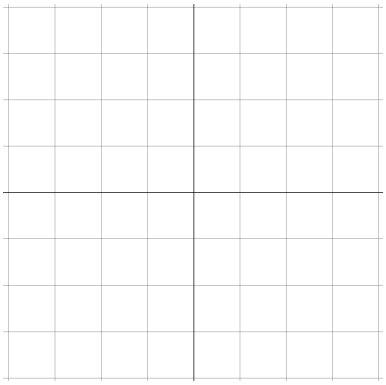


8. Sketch the graph of the following functions. Label all intercepts and asymptotes as appropriate. (13 pts)

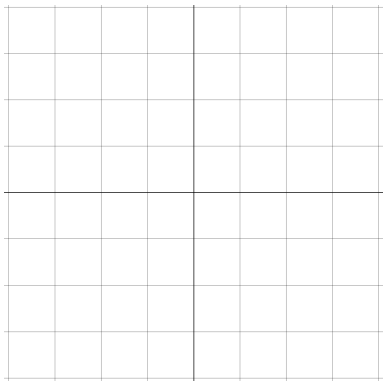
(a) $f(x) = \sqrt{-x} - 1$.



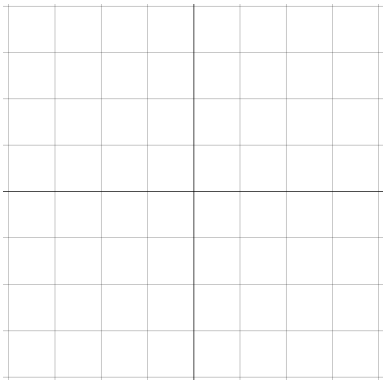
(b) $q(x) = \begin{cases} e^x + 1 & \text{if} \quad x \leq 0 \\ 3 & \text{if} \quad x > 0 \end{cases}$



(c) $h(x) = \cos(x)$ on domain $[0, 2\pi]$



(d) $k(x) = \sec(x)$ for one cycle



9. Given $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x-5}$ answer the following. (8 pts)

(a) Find $(f + g)(6)$.

(b) Find $(f \circ g)(x)$ **and** find the domain.

(c) Find $(g \circ f)(x)$ **and** find the domain.

10. Find the exact value: (18 pts)

(a) $\cos\left(\frac{11\pi}{6}\right)$

(b) $\sin\left(\frac{3\pi}{2}\right)$

(c) $\sec\left(-\frac{4\pi}{3}\right)$

(d) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$

(e) $\tan^{-1}(-1)$

(f) $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

11. Verify the identity: $\sin^2\theta\tan\theta = \tan\theta - \sin\theta\cos\theta$ (5 pts)

(Hint: Start with the right hand side and recall the definition of $\tan\theta$)

12. Find all solutions to the following equations: (10 pts)

(a) $\sqrt{3}\cos\theta - 2\sin\theta\cos\theta = 0$

(b) $\sin\left(\frac{\theta}{3}\right) = \frac{1}{2}$

13. A bird, perched on a sheer cliff, spots a beetle on the flat ground 100 feet away from the base of the cliff. The bird flies straight to the beetle, snatches it up in its beak, and then runs 8 feet along the ground to join its flock. Suppose $30^\circ$ is the angle between the bird's flight path and the ground. From the time the bird took flight, how far in total did the bird travel when it joined its flock? (5 pts)

14. Find the exact value for each: (8 pts)

(a) $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$

(b) $\cos\left(-\frac{\pi}{12}\right)$

15. For $f(x) = 3\sin\left(x - \frac{\pi}{3}\right)$ (8 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $f(x)$. Label the phase shift and ending $x$-value of the cycle on the $x$-axis and amplitude values on the $y$-axis to receive full credit.

**END OF FINAL EXAM. HAVE AN AWESOME SUMMER!**