APPM 1345
Final Exam
Spring 2025

Name	
Instructor	**Lecture Section**

This exam is worth 150 points and has **7 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, you may ask one of your proctors for a piece of scratch paper. Do NOT use any paper that you have brought with you.

Show all work and *simplify* your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End of Exam Check List

1. If you finish the exam before 12:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 12:45 PM:
   - Please wait in your seat until 1:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C \qquad \int \frac{1}{1+x^2}\,dx = \arctan(x) + C$$

1. (20 pts) The following parts are unrelated.

   (a) Find $\dfrac{dy}{dx}$ for $y = \ln\left(7 - 3x^4\right)$.

   (b) Find y' for: $xe^y - \cosh(y) = 399$

2. (36 pts) The following are unrelated.

(a) Evaluate the limit (you may leave your answer in terms of hyperbolic functions): $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{3x}$

(b) Evaluate the limit: $\lim_{x \to 0} \frac{\arcsin(x)}{x}$

(c) Evaluate the definite integral $\int_{1}^{\ln(2)} \frac{5e^x}{e^x + 1}\, dx$

(d) Evaluate the indefinite integral $\int \frac{\sin(\theta)}{1 + \cos^2(\theta)}\, d\theta$

3. (16 pts) Consider the function $f(x) = 5x^2 + 2x - 3$ on the interval $[-1, 2]$.

   (a) Approximate $\int_{-1}^{2} f(x)\, dx$ using two rectangles of equal width with the right end point rule (R_2).

   (b) Show that $f(x)$ satisfies the hypotheses of the Mean Value Theorem.

   (c) Find all numbers, c, that satisfy the conclusion of the Mean Value Theorem.

4. (16 pts) For $f(x) = \int_{2x}^{1} \sin^{-1}(t)\ dt$ answer the following:

(a) Find $f'(x)$.

(b) Find the equation of the line tangent to f that passes through the point $\left(\frac{1}{4}, \frac{5\pi - 6\sqrt{3}}{12}\right)$.

(c) Find $f''(x)$.

5. (18 pts) The position, s, of a particle moving in a straight line is given by $s(t) = \frac{t^3}{3} - \frac{t^2}{2} - 6t$ ft for time t in seconds. The equation is valid for $t \geq 0$. Be sure to include units below where relevant.

(a) Find the velocity of the particle as a function of t.

(b) On what interval(s) of time is the particle moving in the positive direction?

(c) Find the acceleration of the particle after 3 seconds.

(d) Find the average velocity of the particle on the interval $[0, 3]$.

(e) Find the total distance traveled by the particle during the first 4 seconds.

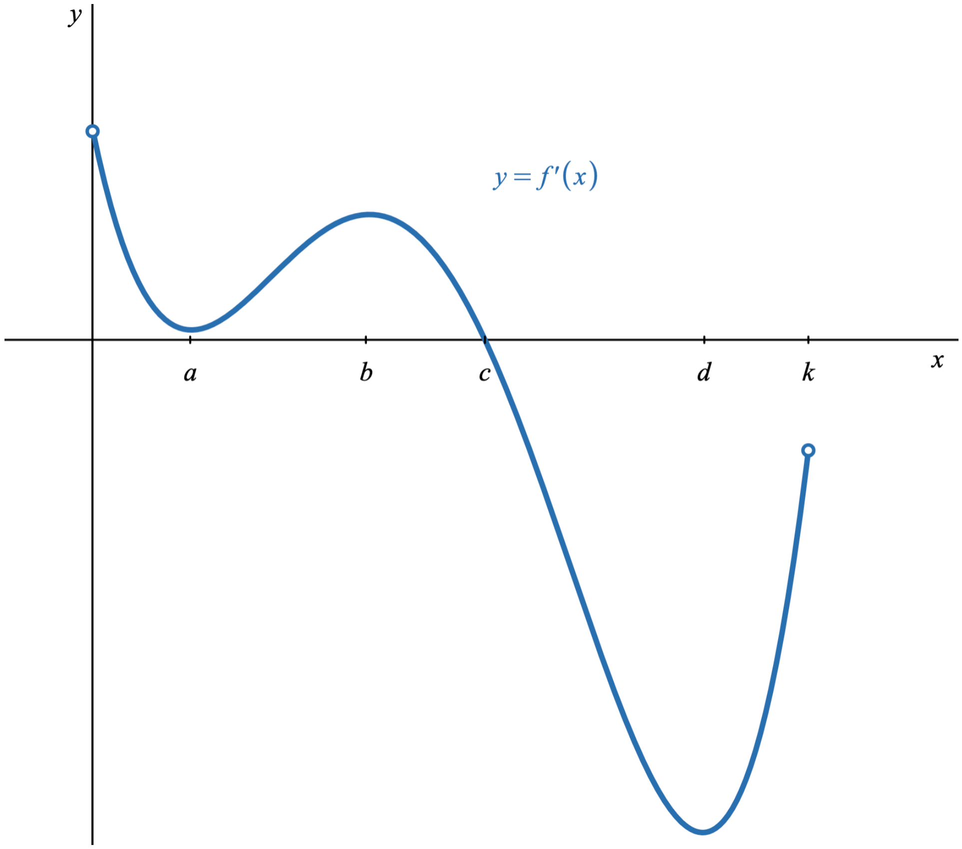
6. (24 pts) The following parts are unrelated.

   (a) A bacteria culture initially contains 106 cells and its population, $P(t)$, grows at a rate proportional to its size. After an hour the population has increased to 420. Find a function for the number of cells after t hours.

   (b) At noon, ship A is 60 km west of ship B. Ship A is sailing south at 15 km/h and ship B is sailing north at 5 km/h. How fast is the distance between the ships changing at 4:00 PM?

7. (20 pts) Parts (a) and (b) below are unrelated.

(a) Below is the graph of the **first derivative**, f', of a function f. Answer the following questions related to f which is defined on $(0, k)$ with x-values: a, b, c, d, and k. (List all answers that apply. Use interval notation where appropriate. No explanation is necessary.)



i. Evaluate $\lim_{h \to 0} \dfrac{f'(b+h) - f'(b)}{h}$

ii. On what intervals is f decreasing?

iii. At what x-value does f have a local maximum?

iv. On what intervals is f concave down?

v. At what value(s) of x does f have an inflection point?

(b) Sketch a graph of a **single function** $y = g(x)$ with **all** of the following properties:

- $g(0) = 1$
- $g(-x) = g(x)$
- $\lim_{x \to 1^-} g(x) = -\infty$
- $\lim_{x \to -1^-} g(x) = +\infty$
- $\lim_{x \to \infty} g(x) = 2$
- $\lim_{x \to -2} g(x) = 3$
- $g'(x) > 0$ if $x < 0$ and $x \neq -1, -2$
- $g(-2)$ DNE

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.