APPM 2350
Final Exam
Spring 2025

Name	
Instructor	Lecture Section

This exam is worth 150 points and has **6 problems**.

Make sure all of your work is written in the blank spaces provided. You can also use the extra space provided at the end of the exam. If after utilizing the extra space at the end of the exam your solutions do not fit, you may ask one of your proctors for a piece of scratch paper. Do NOT use any paper that you have brought with you.

Show all work and ***simplify*** **your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

You are allowed one page of notes (8.5 inches by 11 inches, two-sided), but other notes, papers, calculators, cell phones, and other electronic devices are not permitted on this exam.

End of Exam Check List

1. If you finish the exam before 12:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 12:45 PM:
   - Please wait in your seat until 1:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

1. (27 points) Consider $f(x, y) = x^3 + 4y^2 - 15x + 7$.

   (a) (6 points) Find all critical points of $f(x, y)$.

   (b) (6 points) Classify each of the critical points as a local maximum, local minimum, or saddle point.

   (c) (6 points) Explain why the Extreme Value Theorem guarantees there are points on the ellipse $3x^2 + 2y^2 = 48$ where $f(x, y)$ will obtain an absolute maximum value and an absolute minimum value.

   (d) (9 points) Find the absolute maximum and minimum values of $f(x, y)$ subject to the constraint $3x^2 + 2y^2 = 48$.

2. (30 points) Captain Bonaventura Cavalieri is a pirate who likes to steal acorns from unsuspecting squirrels. He recently stole an acorn from Sam the Squirrel after Sam accidentally dropped it while running in a park. Captain Bonaventura Cavalieri stores these acorns in a vault that occupies the region in space, $\mathcal{E}$, given by $x^2 + y^2 \leq 25$, $x \leq 0$, and $0 \leq z \leq 10$.

   Pam the Penguin learns of this piracy, and goes to the vault. She presses an "eject" button that jettisons all of the acorns from the vault. The movement of the acorns can be described with the velocity field

   $$\mathbf{F}(x, y, z) = \langle z^2 y, x^2 y, y^2 z \rangle.$$

   (a) (15 points) Setup but **do not evaluate** a double integral that gives the outward flux of the acorns through the piece of the surface of the vault given by $x^2 + y^2 = 25$ and $x \leq 0$ for $-5 \leq y \leq 5$ and $0 \leq z \leq 10$. (For full credit, this integral should be fully set-up with correct limits of integration, not be left in terms of any vectors or vector operations, and the integrand should be in terms of only the variables with which one would integrate.)

   (b) (15 points) Compute the outward flux of the acorns through the entire surface of the vault. For full credit, you **will evaluate the integral** in this problem. (Hint: You may want to use one of our major vector calculus (chapter 13) theorems. If you do so, be sure to mention the name of the theorem you use.)

3. (15 points) Pam the Penguin returns an acorn to Sam the Squirrel by walking along the path, $\mathcal{C}$, that goes along the curve $x^2 + y^3 = 8$ from $(2\sqrt{2}, 0)$ to $(0, 2)$. Suppose the force required to move the acorn is given by

$$\mathbf{F}(x,y) = (ye^x + \sin y)\mathbf{i} + (e^x + x\cos y + 2y)\mathbf{j}.$$

Find the work done in moving the acorn along $\mathcal{C}$ **without parameterizing** the curve.

4. (30 points) Upon the return of his long lost acorn, Sam the Squirrel is filled with excitement and starts running around in circles. Specifically, he is running counterclockwise around the circle of radius 2 centered at the origin. The force he uses in running around this circle is given by $\mathbf{F}(x, y) = \langle x^2 + y^2, x \rangle$. We will find the work Sam does in running around this circle once in two different ways:

   (a) (15 points) Parameterize the curve and compute the integral directly.

   (b) (15 points) Compute the integral by applying an appropriate major theorem from vector calculus (chapter 13). Be sure to state the name of the theorem you use.

5. (21 points) Consider the point $P(0, 0, 1)$ and the plane $z = -1$.

   (a) (9 points) Determine the equation of the surface of all points that are equidistant from the point P and the plane mentioned above. Simplify the equation to a standard form presented in class for that type of surface.

   (b) (3 points) Identify the type of surface described in part (a).

   (c) (9 points) Find the equation of the plane that contains the point P and the line $\mathbf{r}(t) = 2t\mathbf{i}+(4t-3)\mathbf{j}+(2-5t)\mathbf{k}$. Write your final answer in the form $ax + by + cz = d$.

6. (27 points) Let $\mathcal{E}$ be the region bounded above by $x^2 + y^2 + z^2 = 17$ and below by $z^2 = 16x^2 + 16y^2$ that lies above the third quadrant of the xy-plane. Consider the integral

$$I = \iiint_{\mathcal{E}} xyz^2 \, dV.$$

(a) (6 points) Make a clear sketch of the cross-section of the solid region in the rz-plane. Assume the constant angle θ lies in the third quadrant of the xy-plane. Axes, intercepts, and curves should be clearly labeled. Shade in the region itself.

(b) (7 points) Express I as an integral or the sum of integrals in cylindrical coordinates using the order $dz\, dr\, d\theta$. Do **NOT** evaluate this integral.

(c) (7 points) Express I as an integral or the sum of integrals in spherical coordinates using the order $d\rho\, d\phi\, d\theta$. Do **NOT** evaluate this integral.

(d) (7 points) Express I as an integral or the sum of integrals in Cartesian coordinates using the order $dz\, dx\, dy$. Do **NOT** evaluate this integral.

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.