- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/031225 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) If $\mathbf{A}$ and $\mathbf{B}$ are both $n \times n$ diagonal matrices, then $\mathbf{AB} = \mathbf{BA}$ always holds.

   (b) If the characteristic polynomial of a 3×3 matrix is $\lambda^3 - 3\lambda^2 - \lambda + 3$, the eigenvalues of the matrix are $1, 3, -3$.

   (c) Let $\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n\}$ be a basis for a vector space $\mathbb{V}$. Then for any other $\vec{\mathbf{y}} \in \mathbb{V}$, $\{\vec{\mathbf{v}}_1, \dots, \vec{\mathbf{v}}_n, \vec{\mathbf{y}}\}$ is also a basis for $\mathbb{V}$.

   (d) For any $m \times n$ matrix $\mathbf{A}$, $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ is consistent if and only if $\vec{\mathbf{b}}$ is in the column space of $\mathbf{A}$, that is, $\vec{\mathbf{b}} \in \text{Col}\,\mathbf{A}$

   (e) If $\mathbf{A}$ and $\mathbf{B}$ are nonsingular matrices, then $\left|(\mathbf{AB})^{-1}\right| = (|\mathbf{A}||\mathbf{B}|)^{-1}$

2. [2360/031225 (23 pts)] Let $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.

   (a) (4 pts) Find the eigenvalues of $\mathbf{A}$ and their algebraic multiplicity.

   (b) (7 pts) Find the geometric multiplicity of the eigenvalue whose algebraic multiplicity from part (a) is one. Find a basis for the eigenspace of this eigenvalue.

   (c) (8 pts) Find a basis for the solution space of $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$. What is the dimension of the solution space?

   (d) (4 pts) Is $\mathbf{A}^{\mathrm{T}}$ invertible? Explain briefly without doing any calculations.

3. [2360/031225 (20 pts)] Consider the linear system
$$\begin{aligned} x_2 + 2x_3 &= 1 \\ 3x_1 + 15x_2 + 6x_3 &= 9\,, \\ 6x_1 + 29x_2 + 10x_3 &= k \end{aligned}$$
where k is a real constant.

   There is a single value of k that makes the system consistent. Find that value and then solve the system using that value for k by finding the RREF of an appropriate matrix. Use the Nonhomogeneous Principle to write the general solution in the form $\vec{\mathbf{x}} = \vec{\mathbf{x}}_h + \vec{\mathbf{x}}_p$, clearly indicating $\vec{\mathbf{x}}_h$ and $\vec{\mathbf{x}}_p$.

MORE PROBLEMS BELOW/ON REVERSE

4. [2360/031225 (16 pts)] Consider the following vectors in $\mathbb{R}^3$:

$$\vec{\mathbf{x}} = \begin{bmatrix} 14 \\ 3 \\ -19 \end{bmatrix}, \vec{\mathbf{u}} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \vec{\mathbf{v}} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \vec{\mathbf{w}} = \begin{bmatrix} 11 \\ 2 \\ -15 \end{bmatrix}$$

Show all your work to justify your answers to the following questions.

(a) (8 pts) Is $\vec{\mathbf{x}} \in \text{span}\ \{\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}\}$?

(b) (8 pts) Is span $\{\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}\} = \mathbb{R}^3$?

5. [2360/031225 (16 pts)] Consider the set $\left\{t^2 + 3t - \frac{5}{4}, -2t^2 + t - 1, \frac{1}{2} - t\right\}$.

(a) (8 pts) Show that the Wronskian cannot be used to decide whether or not the set is linearly independent.

(b) (8 pts) Does the set form a basis for $\mathbb{P}_2$? Justify your answer.

6. [2360/031225 (15 pts)] For each of the following, determine if the given subset, $\mathbb{W}$, is a subspace of the given vector space, $\mathbb{V}$. Assume that standard operations apply in each case. Provide justification for your answers.

(a) (5 pts) $\mathbb{V} = \mathbb{R}^3$; $\mathbb{W}$ is the set of vectors of the form $[n \quad n \quad 2n]^{\mathrm{T}}$ where n is an integer.

(b) (5 pts) $\mathbb{V} = C(-\infty, \infty)$ (the set of functions that are continuous for all real numbers); $\mathbb{W}$ equals the set of all constant functions.

(c) (5 pts) $\mathbb{V} = \mathbb{M}_{nn}$; $\mathbb{W}$ is the set of $n \times n$ matrices with trace equal to -1.