

1. (16 pt) Determine whether the series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{10}{\sqrt[3]{n^2 + 1000}}$

(b) $\sum_{n=1}^{\infty} \frac{3}{(\arctan n)^3}$

2. (22 pt)

- (a) Determine whether the series $\sum_{n=3}^{\infty} (-1)^n \frac{\ln n}{n}$ is absolutely convergent, conditionally convergent, or divergent.

- (b) Determine the interval of convergence, including any endpoints, for the series $\sum_{n=3}^{\infty} \frac{(\ln n)(x-1)^n}{n}$.

3. (18 pt) Let $g(x) = \frac{1}{(1-8x)^2}$.

- (a) Find a power series representation for $g(x)$. Write your answer in sigma notation.

- (b) Use your answer for part (a) to find a power series representation for $\int x^8 g(x) dx$.

4. (18 pt) Let $f(x) = \sin(3x)$. Be sure to simplify your answers to the following problems.

- (a) Find $T_3(x)$, the 3rd degree Taylor polynomial of f , centered at 0, and use it to approximate the value of $\sin(1)$.

- (b) Use Taylor's Formula to find an error bound for the approximation.

5. The following three problems are not related.

- (a) (8 pt) Use the binomial series representation for $(1+4x)^{3/4}$ to find the coefficient of the x^3 term. Fully simplify your answer.

- (b) (8 pt) Find the sum of the series

$$\frac{1}{e} - \frac{1}{3e^3} + \frac{1}{5e^5} - \frac{1}{7e^7} + \cdots$$

- (c) (10 pt) Consider the parametric curve defined by $x = 3 \tan^2 t$, $y = 3 \sec^2 t$ for $0 \leq t \leq \pi/4$.

- Find the x and y coordinates of the initial and terminal points of the curve.
- Eliminate the parameter to find a Cartesian equation for the curve.