APPM 4/5565 - Random Graphs

Course Objectives: This class introduces students to a variety of mathematical techniques to analyze random graph models, particularly the Erdös-Rényi model.

Specifically, students will:

- Learn to use generating functions to count combinatorial objects and study random realizations of these.
- Learn widely applicable probabilistic methods to study random discrete structures.
- Learn in detail various properties of the Erdös-Rényi random graph model.
- Gain basic knowledge of various widely adopted random graph models.

Textbooks:

- Analytic Combinatorics, 1st Edition by P. Flajolet & R. Sedgewick (optional).
- Networks, 1st Edition by M. Newman (optional).
- Introduction to Random Graphs, 1st Edition by A. Frieze & M. Karoński (optional).
- Random Graphs, 1st Edition by S. Janson, T. Luczak, & A. Rucinski (optional).

Prerequisites:

- APPM 3570, MATH 4510, or equivalent.

Topics Covered

- **Generating Functions (10 lectures)**
  - Symbolic Method
  - Probabilistic calculations
  - Lagrange inversion formula
  - Bivariate Generating Functions
- **Asymptotics (6 lectures)**
  - Asymptotic notation
  - Exp-Log transform
  - Chernoff bound

  - Lindeberg-Lévy central limit theorem
  - Martingales central limit theorem
  - Law of rare events
  - The Chen-Stein method
- **Erdös-Rényi Model - PART I (10 lectures)**
  - Degree distribution
  - Threshold functions
  - First-moment method
  - Second-moment method
- **Stochastic Processes (4 lectures)**
  - Galton-Watson branching processes
  - Brownian motion
- **Erdös-Rényi Model - PART II (10 lectures)**
  - Connectivity: sub-critical regime
  - Connectivity: super-critical
  - Connectivity: critical regime
  - Subgraphs
- **Other Random Graph Models (3 Lectures)**
  - Preferential attachment model
  - Stochastic block model
  - Fixed-degree distribution models