APPM 2350

Exam 3

Spring 2025

Name	
Instructor	**Lecture Section**

This exam is worth 100 points and has **6 problems**.

Make sure all of your work is written in the blank spaces provided. You can also use the extra space provided at the end of the exam. If after utilizing the extra space at the end of the exam your solutions do not fit, you may ask one of your proctors for a piece of scratch paper. Do NOT use any paper that you have brought with you.

Show all work and ***simplify*** **your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

You are allowed one page of notes (8.5 inches by 11 inches, one-sided), but other notes, papers, calculators, cell phones, and other electronic devices are not permitted on this exam.

End of Exam Check List

1. If you finish the exam before 7:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 7:45 PM:
   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

1. (11 points) Consider the vector field $\mathbf{F}(x, y, z) = \langle yz, -y^2 z, yz^2 \rangle$.

   (a) (4 points) Find the divergence of the vector field $\mathbf{F}$.

   (b) (4 points) Find the curl of the vector field $\mathbf{F}$.

   (c) (3 points) Is $\mathbf{F}$ incompressible? Briefly explain why or why not.

2. (14 points) Evaluate $\int_0^2 \int_y^2 e^{x^2}\, dx\, dy$. (Hint: You may find it helpful to sketch the region and switch the order of integration.)

3. (19 points) Captain Bonaventura Cavalieri is a pirate who likes to steal acorns from unsuspecting squirrels. He recently stole an acorn from Sam the Squirrel after Sam accidentally dropped it while running in a park. Captain Bonaventura Cavalieri stores these acorns in a vault built on the region of land on the xy-plane, $\mathcal{R}$, bounded by $x^2 + y^2 = 25$ where $x \leq 0$. The mass density of acorns in this vault is given by $\rho(x, y) = y^2$ kilograms per square meter.

   (a) (14 points) Evaluate an integral to determine the mass of acorns in this vault. (Include the correct units in your final answer.)

   (b) (5 points) Find the average value of $\rho(x, y)$ over the region $\mathcal{R}$. (Include the correct units in your final answer.)

4. (17 points) Evaluate the surface integral $\iint_S x\, dS$ where S is the surface $z = x^2 + y$ for $0 \leq x \leq 1$ and $0 \leq y \leq 3$.

5. (18 points) Consider the polygonal region in the xy-plane with vertices $(1/2, 1/2)$, $(1, 1)$, $(1, 0)$, and $(2, 0)$. We will evaluate $\iint_{\mathcal{R}} 8xy\, dA$ by applying the change of variables $u = x + y$ and $v = x - y$. Let us proceed in the following steps:

   (a) (4 points) Find the transformations of the vertices into (u, v) coordinates using the change of variables $u = x + y$ and $v = x - y$.

   (b) (6 points) Sketch the polygonal region $\mathcal{S}$ in the uv-plane formed by the vertices you found in (a). Axes and the coordinates of vertices should be clearly labeled. Shade in the region itself.

   (c) (8 points) Determine the value of $\iint_{\mathcal{R}} 8xy\, dA$ by evaluating the appropriate integral over $\mathcal{S}$.

6. (21 points) The integral $V = \int_0^{2\pi} \int_2^{2\sqrt{3}} \int_2^{\sqrt{16-r^2}} r\,dz\,dr\,d\theta$ provides the volume of a solid region.

(a) (5 points) Make a clear sketch of the cross-section of the solid region in the rz-plane. Axes, intercepts, and curves should be clearly labeled. Shade in the region itself.

(b) (8 points) Express V as the sum of integral(s) in spherical coordinates using the order $d\phi\,d\rho\,d\theta$. Do **NOT** evaluate this integral.

(c) (8 points) Express V as the sum of integral(s) in Cartesian coordinates using the order $dz\,dx\,dy$. Do **NOT** evaluate this integral.

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.