

APPM 1345

Exam 3

Spring 2025

Name		
Instructor	Richard McNamara	Section 150

This exam is worth 100 points and has **4 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End-of-Exam Procedure

1. Go to the designated area to scan and upload your exam to Gradescope.
2. Verify that your exam has been correctly uploaded and all problems have been labeled.
3. Hand the physical copy of your exam to a proctor.
4. Have a proctor swipe your BuffOne card.

Formula

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

1. (17 pts) A population of animals is increasing according to an exponential growth model. The initial population is 100 animals, and the population at $t = 10$ years is 300 animals.
 - (a) What was the animal population at $t = 5$ years? For full credit, express your answer without using any logarithmic or exponential terms.

(b) How long will it take the population to increase from its initial size to a population of 2500 animals? Provide an exact expression for your answer and include the correct unit of measurement.

(c) What was the instantaneous population growth rate (animals per year) at $t = 10$? Simplify your answer.

2. (22 pts) Parts (a) and (b) are not related.

(a) Consider the function $f(x) = \frac{2^x}{5 + 2^x}$.

i. Explain why f is invertible, based on its derivative.

ii. Find the inverse function of $f(x) = \frac{2^x}{5 + 2^x}$ and express it in the form $f^{-1}(x)$.

- (b) Consider the function $g(x) = 4e^{x-1} - 3e^{5-x^2}$, which is invertible on $[0, \infty)$. Find an equation of the line that is tangent to the curve $y = g^{-1}(x)$ at the point $(e, 2)$.

Hint: Do not attempt to explicitly identify the function $g^{-1}(x)$.

3. (25 pts) Parts (a) and (b) are not related.

(a) Use properties of logarithms to find $h'(x)$ for the function $h(x) = \ln \left(\frac{x^5 \sqrt{\cos x}}{(2x^2 + 3x + 9)^3} \right)$.

You do not need to fully simplify your answer.

(b) Find the value of $y'(1)$ for the function $y = (1 + 2x)^{1/x}$.

4. (36 pts) Parts (a), (b), and (c) are not related.

(a) Evaluate $\int \frac{6x + 12}{x^2 + 4x + 1} dx$.

(b) Evaluate $\int \frac{7\sqrt{x}}{\sqrt{x}} dx$

(c) Evaluate $\int_{\pi/6}^{\pi/3} \tan \theta \, d\theta$. Simplify your answer to include only one logarithmic term.

END OF EXAM

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.