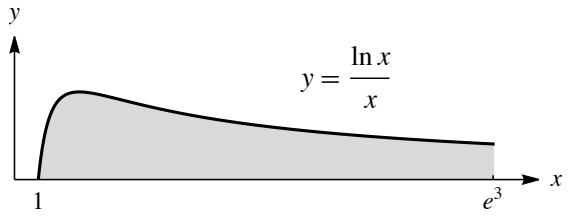
1. Region $\mathcal{R}$ is bounded by the curve $y = (\ln x)/x$ and the x-axis on the interval $\left[1, e^3\right]$.



   (a) (24 pts) Set up (but do not evaluate) integrals to find the following quantities.
      i. The volume of the solid generated by rotating region $\mathcal{R}$ about the y-axis.
      ii. The volume of the solid with $\mathcal{R}$ as the base and cross-sections perpendicular to the x-axis in the shape of squares.
      iii. The area of the surface generated by rotating the given curve $y = (\ln x)/x$ on $\left[1, e^3\right]$ about the line $y = -2$.

   (b) (18 pts) A thin metal plate with constant density ρ covers the same region $\mathcal{R}$ shown. The moment about the y-axis of the plate is $M_y = 3 + 6e^3$. Evaluate integrals to solve the following problems.
      i. Find the value of ρ.
      ii. Find the mass of the plate.

2. (12 pts) Solve the differential equation for y.

$$\sec^4 x \, \frac{dy}{dx} = \frac{\sin^3 x}{y}$$

3. The following parts are not related. Justify all answers.

   (a) (8 pts) Does the sequence $\{m \arctan(5/m)\}$ converge? If so, what does it converge to?

   (b) (14 pts) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{6}{n^2 + 3n + 2}$ or explain why the sum does not exist.

   (*Hint:* Begin with a partial fraction decomposition.)

4. The following parts are not related. Justify all answers.

   (a) (8 pts) Let s_n be the nth partial sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$. Suppose $\lim_{n\to\infty} s_n = 8$.
      i. Find $\lim_{n\to\infty} a_n$.
      ii. Find the sum of the series.

   (b) (8 pts) The nth partial sum of the series $\displaystyle\sum_{n=1}^{\infty} b_n$ is $s_n = 5\left(\frac{2}{5}\right)^n$.
      i. Find the third term of the series.
      ii. Find the sum of the series.

   (c) (8 pts) Find the value of k that satisfies

$$e^{2k} + e^{4k} + e^{6k} + e^{8k} + \cdots = \frac{1}{2}.$$