**APPM 2350**

**Exam 2**

**Spring 2025**

| **Name** | |
|---|---|
| **Instructor** | **Lecture Section** |

This exam is worth 100 points and has **4 problems**.

**Make sure all of your work is written in the blank spaces provided. You can also use the extra space provided at the end of the exam.** If after utilizing the extra space at the end of the exam your solutions do not fit, you may ask one of your proctors for a piece of scratch paper. Do NOT use any paper that you have brought with you.

**Show all work and** ***simplify*** **your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

You are allowed one page of notes (8.5 inches by 11 inches, one-sided), but other notes, papers, calculators, cell phones, and other electronic devices are not permitted on this exam.

**End of Exam Check List**

1. If you finish the exam before 7:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 7:45 PM:
   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

1. For each of the following limits, evaluate it or show that it does not exist.

   (a) (8 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3 - 4y^2}{x^2 + 2y^2}$

   (b) (8 points) $\displaystyle\lim_{(x,y)\to(5,3)} \frac{(2x - y - 2)^2 - (2x + y - 8)^2}{6 - 2y}$

2. Consider $f(x, y) = \ln(y(x-1)^2)$.

(a) (5 points) For which points $(x, y)$ is $f(x, y)$ continuous?

(b) (9 points) Determine the linear approximation (linearization) of $f(x, y)$ centered at $(2, e)$.

(c) (5 points) Use your linear approximation from (b) to approximate $f\left(\frac{39}{20}, \frac{21e}{20}\right)$.

(d) (9 points) Using Taylor's Theorem, what is the maximum possible error when using the linear approximation from (b) to approximate $f(x, y)$ when $|x-2| \leq 0.2$ and $|y-e| \leq 0.2$? (You may find it useful in to recall that $e \approx 2.71$. You should **not** use this for earlier parts of the problem.)

3. The elevation of the ground in a park is given by $g(x, y) = 8xy - \frac{1}{4}(x + y)^4$.

(a) (13 points) Determine the location of all local maximums, local minimums, and saddle points. (Assume the park has no boundary. And, be sure to fully classify each as a local maximum, local minimum, or a saddle point.)

(b) (15 points) Sam the Squirrel is running through the park with a big acorn in his mouth and has location $\mathbf{r}(t) = \langle t^3,\ 3t - t^2 \rangle$ after $t$ minutes.

i. Find the rate of change of Sam's elevation with respect to *time* when he is at the point $(1, 2)$.

ii. Find the rate of change of Sam's elevation with respect to *distance* when he is at the point $(1, 2)$.

iii. Sam accidentally drops his acorn at the point $(1, 2)$ and it rolls in the direction of steepest descent. Find the unit vector in this direction.

4. (28 points) Pam the Penguin has bought a parcel of land that is the shape of a disk with radius 3 kilometers that can be described by the inequality $x^2 + y^2 \leq 9$. The average temperature on this disk, in degrees Celsius, is given by $T(x, y) = 10 + 2x^2 + (y - 1)^2$.

   (a) Pam wants to place a mailbox on the point on the boundary of the disk with the coldest average temperature.

      i. (11 points) Use Lagrange multipliers to determine which point on the boundary she should place her mailbox.

      ii. (2 points) What will the average temperature be at the point you found in i?

   (b) Pam wants to build her home at the point on the disk with the coldest average temperature.

      i. (6 points) Explain why the Extreme Value Theorem guarantees there is a point on this disk with the coldest average temperature.

      ii. (9 points) At what point on the disk should Pam build her home to ensure it has the coldest average temperature possible?

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.