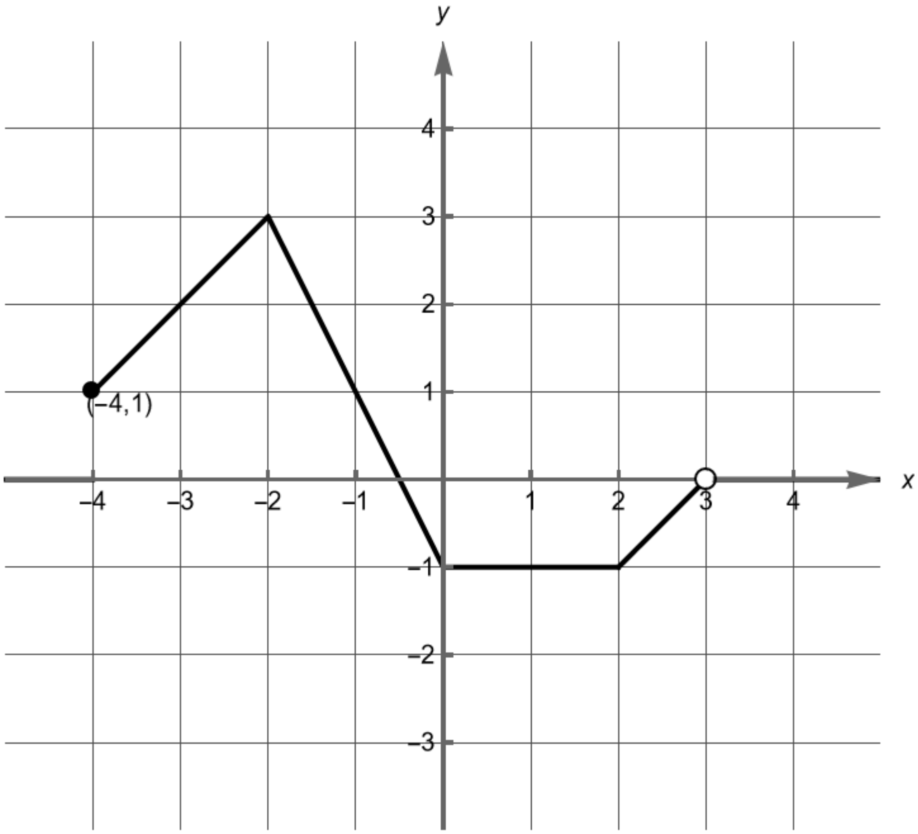
1. Refer to the given graph of $g(x)$ to answer the following: (10 pts)



(a) Find the domain of $g(x)$. Express your answer in interval notation
Solution:
$\boxed{[-4,3)}$

(b) Find the range of $g(x)$. Express your answer in interval notation
Solution:
$\boxed{[-1,3]}$

(c) Find $(g \circ g)(0)$
Solution:
$(g \circ g)(0) = g\left(g(0)\right) = g(-1) = \boxed{1}$

(d) Find $(g+g)(0)$
Solution:
$(g+g)(0) = g(0) + g(0) = -1 + (-1) = \boxed{-2}$

(e) Find x-value(s) for which $g(x) = 3$.
Solution:
$\boxed{x = -2}$

(f) Find x-values for which $g(x) \leq -1$. Give your answer in interval notation.
Solution:
$\boxed{[0,2]}$

(g) Find the net change of $g(x)$ from $x = -3$ to $x = 2$
Solution:
The net change is given by

$$g(2) - g(-3) = -1 - 2 \quad (1)$$
$$= \boxed{-3} \quad (2)$$

(h) Is $g(x)$ odd, even, or neither?
Solution:
$\boxed{\text{Neither, because its not symmetric about either the y axis or the origin.}}$

(i) Identify a restriction of the domain so that g is one-to-one and has the same range as in part (b). Give your answer in interval notation.
Solution:
$\boxed{[-2, 0]}$

(j) Use your domain restriction to calculate $g^{-1}(1)$.
Solution:
We notice that $g(-1) = 1$. Hence $\boxed{g^{-1}(1) = -1}$

2. For the function $f(t) = \dfrac{t-1}{3}$: (7 pts)

(a) Find $(f \circ f)(t)$
Solution:

$$(f \circ f)(t) = \frac{\frac{t-1}{3} - 1}{3} \tag{3}$$
$$= \frac{\frac{t-1}{3} - \frac{3}{3}}{3} \tag{4}$$
$$= \frac{\frac{t-4}{3}}{3} \tag{5}$$
$$= \boxed{\frac{t-4}{9}} \tag{6}$$

(b) Find the domain of $(f \circ f)(t)$
Solution:
$\boxed{(-\infty, \infty)}$

3. For $f(x) = -x^2 + 1$ find the following: (7 pts)

(a) $f(a)$
Solution:
$\boxed{-a^2 + 1}$

(b) $f(a+h)$
Solution:
$\boxed{-(a+h)^2 + 1 = -a^2 - 2ah - h^2 + 1}$

(c) $\dfrac{f(a+h) - f(a)}{h}$
Solution:

$$\frac{f(a+h) - f(a)}{h} = \frac{-(a+h)^2 + 1 - (-a^2 + 1)}{h} \tag{7}$$
$$= \frac{-(a^2 + 2ah + h^2) + 1 - (-a^2 + 1)}{h} \tag{8}$$
$$= \frac{-a^2 - 2ah - h^2 + 1 + a^2 - 1}{h} \tag{9}$$
$$= \frac{-2ah - h^2}{h} \tag{10}$$
$$= \boxed{-2a - h} \tag{11}$$

4. A chemist starts heating some liquid in a test tube. The temperature of the liquid depends on time and grows linearly from 35° Celsius at time $t = 3$ seconds to 42° Celsius at time $t = 16$ seconds. Answer the following questions (7 pts):

   (a) Find the linear equation that relates the temperature T and the time t.

   Solution:
   We will model the temperature T as a linear function of time t where the slope is given by

$$\frac{T(16) - T(3)}{16 - 3} = \frac{42 - 35}{13} \quad (12)$$

$$= \frac{7}{13} \quad (13)$$

   Now the equation of the straight line in point-slope form, using the point $(3, 35)$ would be

$$T - 35 = \frac{7}{13}(t - 3) \quad (14)$$

$$T = \boxed{\frac{7}{13}(t - 3) + 35} \quad (15)$$

   (b) What does the slope of the line from part (a) represent physically?
   Solution:
   The slope of the line from part (a) represents $\boxed{\text{the average rate of change of temperature with time.}}$

   (c) What does the T-intercept of the line from part (a) represent physically?
   Solution.
   the T-intercept of the line from part (a) represents $\boxed{\text{the temperature of the liquid just before}}$ $\boxed{\text{the chemist starts heating the liquid, at time } t = 0}$

5. Find the center and radius for: $x^2 + y^2 - 4y = 3$. (5 pts)
   Solution:
   We find the center and radius by completing the square

$$x^2 + y^2 - 4y = 3 \quad (16)$$

$$x^2 + y^2 - 4y + 4 = 3 + 4 \quad (17)$$

$$x^2 + (y - 2)^2 = 7 \quad (18)$$

$$(x - 0)^2 + (y - 2)^2 = (\sqrt{7})^2 \quad (19)$$

   Hence the center of the circle is $\boxed{(0, 2)}$ and the radius is $\boxed{\sqrt{7}}$

6. The following are unrelated: (10 pts)

   (a) Find the equation of the line that is parallel to the x-axis and passes through the point $(-2, 5)$.

   Solution:
   Since the line is parallel to the x-axis, its y coordinate never changes. As it passes through the point $(-2, 5)$, the equation of the line is $\boxed{y = 5}$

   (b) $g(x)$ is an even function with domain $(-\infty, \infty)$. The point $(3, 5)$ lies on its graph. Which of the following points also lies on its graph?
   (i) $(-3, 5)$ (ii) $(-3, -5)$ (iii) $(3, -5)$ (iv) None of these
   Solution:
   $\boxed{(-3, 5)}$

(c) Find all value(s) of b such that the distance between the two points, $(0, 2)$ and $(1, b)$, is 2.
Solution:
Using the distance formula between 2 pts, we have

$$\sqrt{(1-0)^2 + (b-2)^2} = 2 \quad (20)$$
$$\sqrt{1 + (b-2)^2} = 2 \quad (21)$$
$$1 + (b-2)^2 = 4 \quad (22)$$
$$(b-2)^2 = 3 \quad (23)$$
$$b - 2 = \pm\sqrt{3} \quad (24)$$
$$b = \boxed{2 \pm \sqrt{3}} \quad (25)$$

(d) If $h(x) = \sqrt{2-x}$ and $k(x) = \sqrt{x-2}$ then what is the domain of $g(x) = (h+k)(x)$?
Solution:
Since we are not allowed to take square roots of negative numbers, the domain of $h(x)$ is $2 \leq x$ and the domain of $k(x)$ is $x \geq 2$. The only way to satisfy both conditions is $x = 2$. Hence the domain of $g(x) = (h+k)(x)$ is given by $\boxed{x = 2 \text{ or } [2, 2]}$.

7. For $h(x) = \dfrac{2}{2x+5}$ answer the following (7 pts):

(a) Find the inverse function for $h(x)$
Solution:
In order to find the inverse, we first call $h(x) = y = \dfrac{2}{2x+5}$. Then we interchange x and y and solve for y in terms of x

$$x = \frac{2}{2y+5} \quad (26)$$
$$2y + 5 = \frac{2}{x} \quad (27)$$
$$2y = \frac{2}{x} - 5 \quad (28)$$
$$y = \frac{1}{x} - \frac{5}{2} \quad (29)$$

Hence the inverse is $\boxed{h^{-1}(x) = \frac{1}{x} - \frac{5}{2}}$.

(b) What is the range of $h(x)$?
Solution:
The range of $h(x)$ is the domain of $h^{-1}(x)$, which is $\boxed{(\infty, 0) \cup (0, \infty)}$.

8. If the equation of a parabola in standard form is given by $y = a(x-3)^2 + 2$ and the parabola passes through the point $(-3, 1)$ find the value of a. (4 pts)
Solution:
Since the parabola passes through $(-3, 1)$, we can write

$$1 = a(-3-3)^2 + 2 \quad (30)$$
$$1 = a(-6)^2 + 2 \quad (31)$$
$$1 = 36a + 2 \quad (32)$$
$$-1 = 36a \quad (33)$$
$$a = \boxed{-\frac{1}{36}} \quad (34)$$

9. Find the domain of the following functions. Express your answers in interval notation. (12 pts)

(a) $n(x) = \dfrac{x^2 - 1}{x^2 - 2x - 3}$

Solution:
We must exclude all x values that make the denominator 0

$$x^2 - 2x - 3 = 0 \tag{35}$$
$$(x + 1)(x - 3) = 0 \tag{36}$$

Hence we domain require that $x \neq -1, 3$. So the domain is $\boxed{(-\infty, -1) \cup (-1, 3) \cup (3, \infty)}$

(b) $h(x) = \dfrac{x\sqrt{3 - x}}{3 + x}$

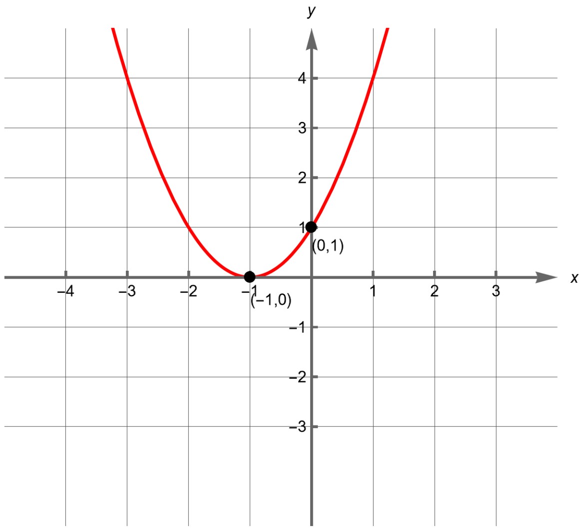
Solution:
Since the square root of a negative quantity does not exist in the real numbers, and the denominator of a fraction can't be 0, we require that both $3 - x \geq 0$ and $x \neq -3$. Hence the domain is $\boxed{(-\infty, -3) \cup (-3, 3]}$

(c) $s(x) = 2x^2 + \sqrt[3]{-x}$

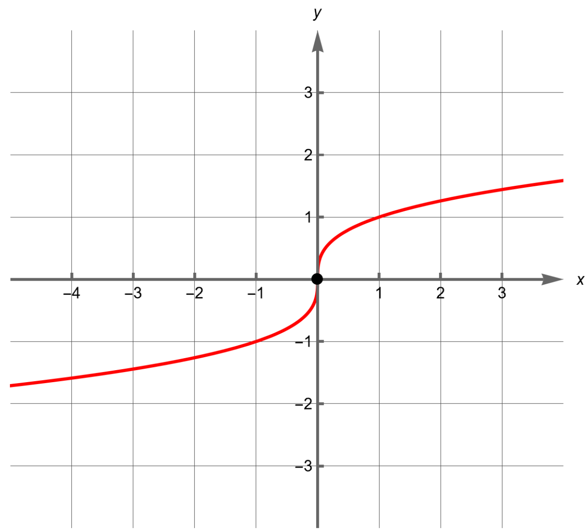
Solution:
The first term is a polynomial and the second is a cube root function, both of which are valid in all real numbers. Hence the domain is $\boxed{(-\infty, \infty)}$

10. Sketch the shape of the graph of each of the following on the provided axes. **Label** at least one value on the x-axis and y-axis for each part. (14 pts)

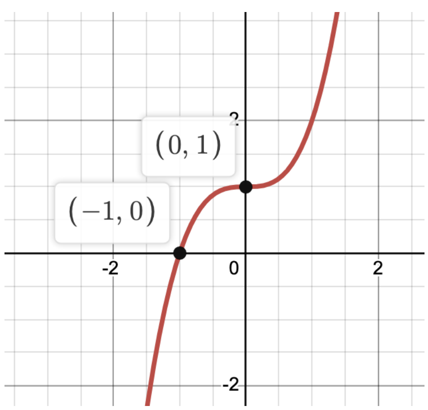
(a) $h(x) = (x+1)^2$



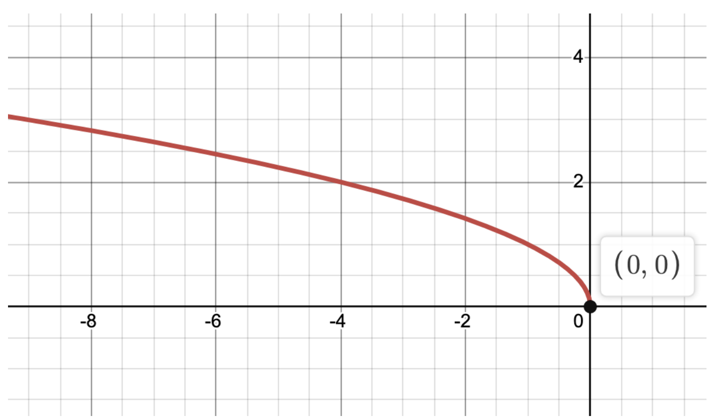
(d) $g(x) = \sqrt[3]{x}$



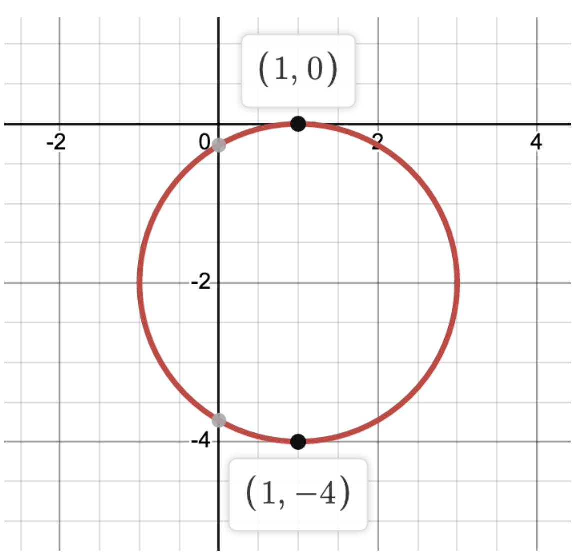
(b) $k(x) = x^3 + 1$



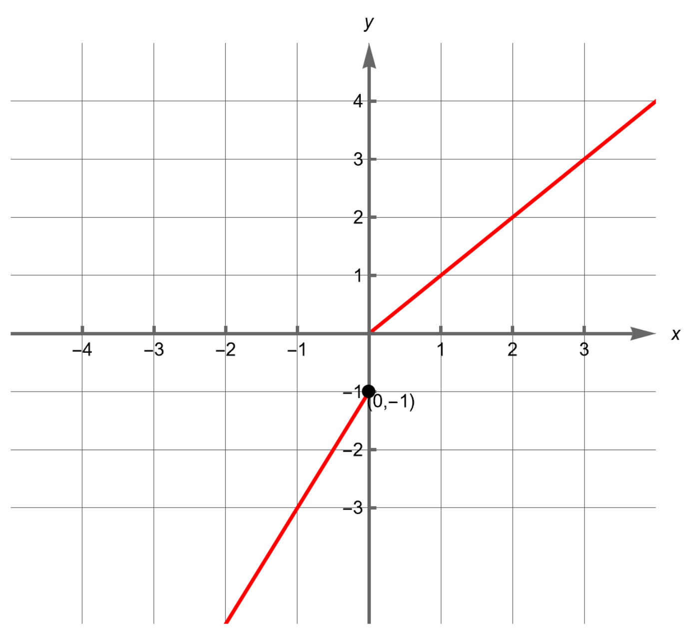
(e) $m(x) = \sqrt{-x}$



(c) $(x-1)^2 + (y+2)^2 = 4$



(f) $q(x) = \begin{cases} 2x-1 & \text{if } x \le 0 \\ |x| & \text{if } x > 0 \end{cases}$



11. For $P(x) = -x^4 + 4x^3 - 3x^2$ answer the following. (11 pts)

(a) i. Identify the term that dominates the end behavior of $P(x)$:

Solution:

The term $\boxed{-x^4}$

ii. Based on your answer to part (i) fill in the blanks for $P(x)$:

Solution:

$y \to \boxed{-\infty}$ as $x \to -\infty$ and $y \to \boxed{-\infty}$ as $x \to \infty$.

(b) Find all zeros of $P(x)$ **and** identify the multiplicity of each zero

Solution:

We find the zeros by solving

$$-x^4 + 4x^3 - 3x^2 = 0 \quad (37)$$

$$-x^2(x^2 - 4x + 3) = 0 \quad (38)$$

$$-x^2(x-3)(x-1) = 0 \quad (39)$$

Hence the zeros are:

$\boxed{x = 0}$ with a multiplicity of 2

$\boxed{x = 1}$ with a multiplicity of 1

$\boxed{x = 3}$ with a multiplicity of 1

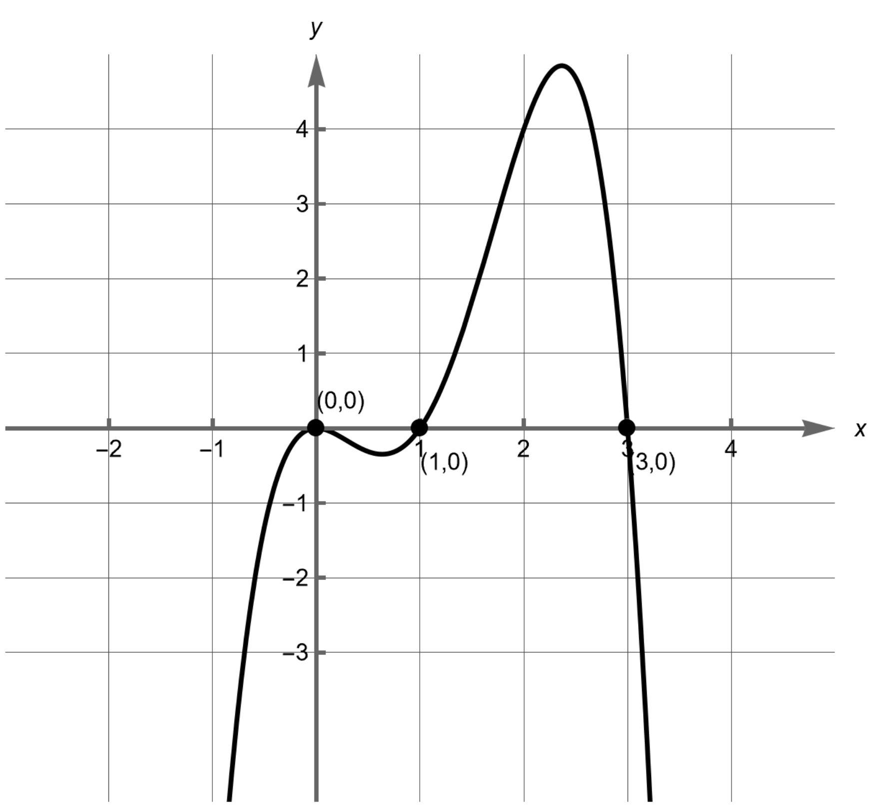
(c) Find the y-intercept.

Solution:

the y-intercept is found by setting x to 0. We notice that the y-intercept is $(0, 0)$

(d) Sketch the graph of $P(x)$ by using all the above information. **Label** all intercepts on the graph.

Solution:



12. A wire 10 cm long is cut into two pieces. Each piece is bent into the shape of a square (one square has side length x and the other square has side length y). Express the total area of the two squares as a function of x. (6 pts)

Solution:
A square of side length x has area x^2, and a square of side length y has area y^2. Hence the total area of the 2 squares is given by

$$A = x^2 + y^2 \tag{40}$$

Now,a square of side length x has perimeter $4x$, and a square of side length y has perimeter $4y$. Since the total length of the original wire was 10, we can write

$$\begin{align} 4x + 4y &= 10 \tag{41} \\ 4y &= 10 - 4x \tag{42} \\ y &= \frac{10 - 4x}{4} \tag{43} \\ &= \frac{5 - 2x}{2} \tag{44} \end{align}$$

Subsituting this value of y in the first equation, we obtain

$$\begin{align} A(x) &= x^2 + \left(\frac{5 - 2x}{2}\right)^2 \tag{45} \\ &= \boxed{x^2 + \frac{1}{4}(5 - 2x)^2} \tag{46} \end{align}$$

END OF EXAM