**APPM 3570/STAT 3100 Exam 1 Spring 2025**

NAME: ______________________________

SECTION: (Circle One) 001 at 10:10 am *or* 002 at 2:30 pm

**Instructions:**

1. Calculators are permitted.
2. Notes, your text and other books, cell phones, and other electronic devices are not permitted—except for calculators or as needed to view and upload your work.
3. Justify your answers, show all work.
4. When you have completed the exam, go to the uploading area in the room and scan your exam and upload it to Gradescope.
5. Don't forget to scan any pages you used for extra space!
6. Verify that everything has been uploaded correctly and the pages have been associated to the correct problems.
7. Turn in your hardcopy exam.

On my honor as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work.

Signature: ______________________ Date: ____________



## Duration: 90 minutes

**Problem 1.** (30 points.) Suppose events $E, F$ and $G$, all defined on the same sample space, have the following probabilities:
$\mathrm{P}(E) = 0.22,\ \mathrm{P}(F) = 0.25,\ \mathrm{P}(G) = 0.28,\ \mathrm{P}(E \cap F) = 0.11,\ \mathrm{P}(E \cap G) = 0.05,\ \mathrm{P}(F \cap G) = 0.07$, and $\mathrm{P}(E \cap F \cap G) = 0.01$. For each of the following questions, your answer should be in the form of a complete mathematical statement.

(a) Let $D$ be the event that at least one of $E, F$ and $G$ occurs. Describe $D$ using set notation and a Venn dagram. Find $\mathrm{P}(D)$.

(b) Let $H$ be the event that exactly one of $E, F$ and $G$ occurs. Describe $H$ using set notation and a Venn dagram. Find $\mathrm{P}(H)$.

(c) What is the probability that event $E$ will occur and event $F$ will not occur?

(d) Given that event $E$ occurs, compute the probability that event $F$ will occur.

(e) Given that at least one of the three events occurs, compute the probability that all three events will occur.

**Solution:**

(a) (6 points.)
$D = E \cup F \cup G$.

$$\begin{aligned}\mathrm{P}(D) &= \mathrm{P}(E \cup F \cup G)\\ &= \mathrm{P}(E) + \mathrm{P}(F) + \mathrm{P}(G) - \mathrm{P}(EF) - \mathrm{P}(EG) - \mathrm{P}(FG) + \mathrm{P}(EFG)\\ &\qquad \text{by the Inclusion-Exclusion Identity}\\ &= .22 + .25 + .28 - .11 - .05 - .07 + .01\\ &= .53\end{aligned}$$

Must show Venn Diagram.

(b) (6 points.)
$H = EF^CG^C \cup E^CFG^C \cup E^CF^CG$
$\mathrm{P}(H) = \mathrm{P}(EF^CG^C) + \mathrm{P}(E^CFG^C) + \mathrm{P}(E^CF^CG)$ by Axiom 3, since the events are disjoint
$\mathrm{P}(EF^CG^C) = \mathrm{P}(E) - \mathrm{P}(EF) - \mathrm{P}(EG) + \mathrm{P}(EFG) = .22 - .11 - .05 + .01 = .07$
$\mathrm{P}(E^CFG^C) = \mathrm{P}(F) - \mathrm{P}(EF) - \mathrm{P}(FG) + \mathrm{P}(EFG) = .25 - .11 - .07 + .01 = .08$
$\mathrm{P}(E^CF^CG) = \mathrm{P}(G) - \mathrm{P}(EG) - \mathrm{P}(FG) + \mathrm{P}(EFG) = .28 - .05 - .07 + .01 = .17$
$\mathrm{P}(H) = .07 + .08 + .17 = .32$

Must show Venn Diagram.

(c) (6 points.)
$\mathrm{P}(EF^C) = \mathrm{P}(E) - \mathrm{P}(EF) = .22 - .11 = .11$

(d) (6 points.)
$\mathrm{P}(F|E) = \frac{\mathrm{P}(EF)}{\mathrm{P}(E)} = \frac{.11}{.22} = .5$

(e) (6 points.)
$\mathrm{P}(EFG|D) = \frac{\mathrm{P}(EFGD)}{\mathrm{P}(D)} = \frac{\mathrm{P}(EFG)}{\mathrm{P}(D)} = \frac{.01}{.53} = \frac{1}{53}$.

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**Problem 2.** (21 points.) There are three unrelated parts to this question.

(a) There are 5 cars to be displayed in 5 parking spaces with all the cars facing the same direction. Of the 5 cars, 3 are red, 1 is blue and 1 is yellow. If the cars are identical except for color, how many different display arrangements of the 5 cars are possible?

(b) A restaurant accepts only Visa and American Express credit cards. If 27% of its customers carry American Express, and 13% carry both cards and 72% carry at least one card, what percentage of its customers carry a Visa card?

(c) In how many ways can 12 identical donuts be distributed among 5 people such that each person will get at least one donut?

**Solution:**

(a) (7 points.)
This is the number of ways to arrange RRRBY. It is $\binom{5}{3,1,1} = \frac{5!}{3!} = 20$.

(b) (7 points.)
Consider the events $V :=$ the event that a customer carries Visa and $A :=$ the event that a customer carries American Express.

$$\begin{aligned} P(V \cup A) &= P(V) + P(A) - P(VA) \\ .72 &= P(V) + .27 - .13 \\ P(V) &= .72 - .27 + .13 \\ &= .58 \end{aligned}$$

(c) (7 points.) $\binom{n-1}{r-1} = \binom{12-1}{5-1} = \binom{11}{4} = 330$

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**Problem 3.** (12 points.) Each of 2 balls is painted either black or gold and then placed in an urn. Suppose that each ball is colored black with probability $\frac{1}{2}$ and that these events are independent.

(a) Suppose that you draw one ball from the urn and it is painted gold. What is the probability that both balls are gold?

(b) Consider a new set of circumstances, unrelated to part (a). Suppose that you are told that at least one of the two balls is painted gold. What is the probability that both balls are painted gold, given that at least one ball is painted gold?

**Solution:**

(a) (6 points.) Let $G_i :=$ the event that the $i$th ball is gold for $i = 1, 2$.
$\mathrm{P}(G_2|G_1) = \mathrm{P}(G_2) = \frac{1}{2}$.

(b) (6 points.) $\mathrm{P}(G_1G_2|\text{“at least one is gold”}) = \mathrm{P}(G_1G_2|G_1G_2^C \cup G_1^C G_2 \cup G_1G_2) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$.



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**Problem 4.** (18 points.) Suppose you need to come up with a password that is 8 letters in length. The only letters that are available to choose from are the letters A, B and C.

(a) If only one letter may be used in a password, how many passwords can be formed?

(b) If exactly two letters must be used in a password, how many passwords can be formed?

(c) If you must use each of the letters at least once, how passwords can be formed?

**Solution:**

(a) (5 points.) There are three, AAAAAAAA, BBBBBBBB, and CCCCCCCC.

(b) (8 points.) If two letters are used and any combination of the two letters are allowed, then there are $2^8$ passwords that can be formed. Included in this count are two passwords that have only one letter. So only $2^8 - 2 = 254$ have two letters. And there are $\binom{3}{2}$ ways to select two letters. Therefore, the answer is $\binom{3}{2} \cdot (2^8 - 2) = 3 \cdot 254 = 762$.

(c) (5 points.) The total number of passwords of length 8 available when using A, B and C is $3^8$. Then, exclude the passwords with only one letter and exclude the passwords with exactly two letters.
$3^8 - 3 - \binom{3}{2} \cdot (2^8 - 2) = 5796$.



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**Problem 5.** (19 points.) Consider the following cumulative distribution function $F(x)$, for the random variable $X$.

$$F(x) = \begin{cases} 0 & \text{if } \ x < -1 \\ \frac{1}{4} & \text{if } \ -1 \leq x < 0 \\ \frac{3}{8} & \text{if } \ 0 \leq x < \frac{1}{2} \\ \frac{3}{4} & \text{if } \ \frac{1}{2} \leq x < 2 \\ 1 & \text{if } \ x \geq 2 \end{cases}$$

(a) Find $P(X = \frac{1}{2})$.

(b) Find $P(-1 < X \leq 2)$.

(c) Find $P(\frac{1}{4} \leq X < 4)$.

**Solution:**

(a) (6 points.)

$$P\left(X = \frac{1}{2}\right) = F\left(\frac{1}{2}\right) - P\left(X < \frac{1}{2}\right) = F\left(\frac{1}{2}\right) - \lim_{n\to\infty} F\left(\frac{1}{2} - \frac{1}{n}\right) = \frac{3}{4} - \frac{3}{8} = \frac{3}{8}$$

(b) (6 points.)

$$P\left(-1 < X \leq 2\right) = P\left(X \leq 2\right) - P\left(X \leq -1\right) = F\left(2\right) - F\left(-1\right) = 1 - \frac{1}{4} = \frac{3}{4}$$

(c) (7 points.)

$$P\left(\frac{1}{4} \leq X < 4\right) = P\left(X < 4\right) - P\left(X < \frac{1}{4}\right) = \lim_{n\to\infty} F\left(4 - \frac{1}{n}\right) - \lim_{n\to\infty} F\left(\frac{1}{4} - \frac{1}{n}\right) = 1 - \frac{3}{8} = \frac{5}{8}$$



**(Use the back if additional space is needed!)**