

1. (37 pts) Evaluate the following integrals and simplify your answers.

(a) $\int \frac{x}{x^2 + 4x + 3} dx$

(b) $\int \frac{\sqrt{x^2 - 9}}{x} dx$

(c) $\int_0^1 \frac{dx}{x(1 + (\ln x)^2)}$

2. (10 pts) Determine whether $\int_{106}^{\infty} \frac{x^3}{\sqrt{x^{10} + \pi}} dx$ is convergent or divergent. Fully explain your reasoning.

3. (38 pts) Consider the integral $I = \int_0^{\pi} x \cos(x/2) dx$.

(a) Estimate the value of I using the trapezoidal approximation T_3 . Fully simplify your answer.

(b) Estimate the error for the approximation T_3 . Express your answer in terms of π and simplify.

(c) Find the exact value of the integral $I = \int_0^{\pi} x \cos(x/2) dx$.

(d) Consider the region bounded by the curve $y = x \cos(x/2)$ and the x -axis on $[0, \pi]$. Suppose the region is rotated about the line $y = 2$ (which lies above the region). Set up (but do not evaluate) an integral to find the volume of the generated solid.

4. (15 pts) Consider the region $\mathcal{R}$ bounded by the curves $x = y^2 - 2$ and $y = \ln x$, between the lines $y = -1$ and $y = 1$.

(a) Sketch and shade the region $\mathcal{R}$. Clearly label each function and all intercepts. (*Hint:* The functions intersect below the line $y = -1$.)

(b) Evaluate an integral to find the area of region $\mathcal{R}$. (*Hint:* Integrate with respect to y .)