APPM 2350
Exam 1
Spring 2025

Name	
Instructor	**Lecture Section**

This exam is worth 100 points and has **5 problems**.

Make sure all of your work is written in the blank spaces provided. You can also use the extra space provided at the end of the exam. If after utilizing the extra space at the end of the exam your solutions do not fit, you may ask one of your proctors for a piece of scratch paper. Do NOT use any paper that you have brought with you.

Show all work and ***simplify*** **your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

You are allowed one page of notes (8.5 inches by 11 inches, one-sided), but other notes, papers, calculators, cell phones, and other electronic devices are not permitted on this exam.

End of Exam Check List

1. If you finish the exam before 7:45 PM:
   - Go to the designated area to scan and upload your exam to Gradescope.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.
2. If you finish the exam after 7:45 PM:
   - Please wait in your seat until 8:00 PM.
   - When instructed to do so, scan and upload your exam to Gradescope at your seat.
   - Verify that your exam has been correctly uploaded and all problems have been labeled.
   - Leave the physical copy of the exam with your proctors in the correct pile for your Lecture Section.

1. (15 points) Consider $\mathbf{a} = 4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 7\mathbf{k}$, and $\mathbf{c} = \mathbf{j} + \mathbf{k}$.

   (a) For each of the following, either compute it, or, if it is not possible to compute, state "NOT POSSIBLE."

      i. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

      ii. $(\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$

      iii. $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$

      iv. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

   (b) For which of the values that you found in (a) can you ascribe a *simple* geometric meaning in the context of the given vectors? What is this *simple* geometric meaning?

2. (12 points) Consider the curves parameterized by $\mathbf{r_1}(t) = \langle 4t, 2-t, t^2+1 \rangle$ and $\mathbf{r_2}(s) = \langle s^2, s^2-3, s \rangle$. Be sure to fully justify your reasoning for each of the following.

(a) Do these two curves intersect? If so, at what point?

(b) If these curves represent the locations of two particles at time, t (or s), do the two particles collide? If so, at what time and at what point? If not, explain why they do not collide.

3. (23 points) Consider the point $P(2,4,2)$ and the line L_1 given by

$$x = 1 + t, \qquad y = 3 \qquad z = t.$$

(a) Determine the coordinates of the point, Q, on L_1 when $t = 0$.

(b) Find the vector projection of $\mathbf{u} = \langle 2, 0, 1 \rangle$ onto $\overline{PQ}$.

(c) Find the symmetric equations of the line, L_2, which passes through P and Q.

(d) Find the acute angle between lines L_1 and L_2.

4. (22 points) Consider the surface given by

$$x^2 + cy^2 + z^2 - 8z = 0$$

where c is a constant.

(a) i. For which value(s) of c will this surface be a sphere?
   ii. When this surface is a sphere, what is its center and radius?

(b) For which value(s) of c, if any, will this surface be:

(i) an ellipsoid? (ii) a cone? (iii) a hyperboloid of one sheet? (iv) a hyperboloid of two sheets?

(v) an elliptic paraboloid? (vi) a hyperbolic paraboloid? (vii) a cylinder?

(c) Suppose $c = 2$. Describe the intersection of this surface with the yz-plane with an equation on the yz-plane.

5. (28 points) A squirrel runs up a tree along a path with the squirrel's position given by $\mathbf{r}(t) = 4\sin t\mathbf{i} - 4\cos t\mathbf{j} + 3t\mathbf{k}$, where t is the number of seconds since the squirrel started running.

   (a) Determine the rate of change of the direction (magnitude of the normal component of acceleration) of the squirrel for an arbitrary time t. .

   (b) Determine the curvature of the squirrel's path for an arbitrary time t.

   (c) How far has the squirrel traveled from $t = 0$ to $t = 2\pi$? (We want the length of the path traveled, not the squirrel's displacement.)

   (d) Find the equation of the osculating plane (formed by $\mathbf{T}$ and $\mathbf{N}$) when $t = \pi/2$. Your final answer should be in the form $ax + by + cz + d = 0$.

END OF TEST

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.