1. The following are unrelated: (9 pts)

   (a) Arrange the following numbers in ascending (increasing) order

   $\{-3^0, (-3)^0, 4^{-2}, (3-\pi), 0\}$

   Solution:

   The numbers in ascending order are:

   $\boxed{\{-3^0, (3-\pi), 0, 4^{-2}, (-3)^0\}}$

   (b) Express the quantity without using absolute value.

   i. $\left|\sqrt{3}-3\right|$

   Solution:

   Since $3 > \sqrt{3}$, $\sqrt{3}-3 < 0$, so $\boxed{\left|\sqrt{3}-3\right| = -\left(\sqrt{3}-3\right) = 3-\sqrt{3}}$

   ii. $|4-x|$ when $x < 4$

   Solution:

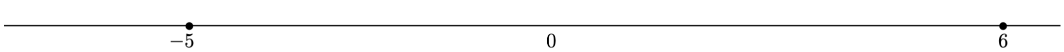
   Since $x < 4$, $4 - x > 0$, so $\boxed{|4-x| = 4-x}$

2. The following are unrelated: (14 pts)

   (a) Use the two values, $\mathbf{-5}$ and $\mathbf{6}$, to answer the following.

   i. Graph the two values on the real number line.

   Solution:



   ii. Find the distance between the two values.

   Solution:

$$
\begin{aligned}
d(-5,6) &= |-5-(6)| && (1)\\
&= |-11| && (2)\\
&= \boxed{11} && (3)
\end{aligned}
$$

(b) Simplify: $\dfrac{|-6-5|+|2|}{3|-3|}$

Solution:

$$\frac{|-6-5|+|2|}{3|-3|} = \frac{11+2}{9} \tag{4}$$
$$= \boxed{\frac{13}{9}} \tag{5}$$

(c) Add and simplify: $\dfrac{2}{\frac{3}{7}} - \dfrac{1}{5} - 8^0$

Solution:

$$\frac{2}{\frac{3}{7}} - \frac{1}{5} - 8^0 = \frac{14}{3} - \frac{1}{5} - 1 \tag{6}$$
$$= \frac{70}{15} - \frac{3}{15} - \frac{15}{15} \tag{7}$$
$$= \boxed{\frac{52}{15}} \tag{8}$$

3. The following are unrelated: (22 pts)

(a) Simplify (give your answer without negative exponents): $\dfrac{4x^{-2}y^3z^{-1}}{14(x^3y^2)^2}$

Solution:

$$\frac{4x^{-2}y^3z^{-1}}{14(x^3y^2)^2} = \frac{2x^{-2}y^3z^{-1}}{7x^6y^4} \tag{9}$$
$$= \frac{2x^{-8}y^{-1}z^{-1}}{7} \tag{10}$$
$$= \boxed{\frac{2}{7x^8yz}} \tag{11}$$

(b) Evaluate the expression: $\sqrt{27}\sqrt{3}$

Solution:

$$\sqrt{27}\sqrt{3} = \sqrt{27.3} \tag{12}$$
$$= \sqrt{81} \tag{13}$$
$$= \boxed{9} \tag{14}$$

(c) Simplify the expression: $\sqrt{4x^2+4}$

Solution:

$$\sqrt{4x^2+4} = \sqrt{4(x^2+1)} \tag{15}$$
$$= \sqrt{4}\sqrt{x^2+1} \tag{16}$$
$$= \boxed{2\sqrt{x^2+1}} \tag{17}$$

(d) Multiply and simplify: $\sqrt[3]{x}\left(\frac{1}{x^{1/3}}+\frac{x^{2/3}}{2}\right)$

Solution:

$$\sqrt[3]{x}\left(\frac{1}{x^{1/3}}+\frac{x^{2/3}}{2}\right) = x^{\frac{1}{3}}\left(\frac{1}{x^{1/3}}+\frac{x^{2/3}}{2}\right) \tag{18}$$
$$= 1+\frac{x^{\frac{1}{3}}x^{\frac{2}{3}}}{2} \tag{19}$$
$$= \boxed{1+\frac{x}{2}} \tag{20}$$

(e) Simplify: $\sqrt[3]{27x^6y^3}$

Solution:

$$\sqrt[3]{27x^6y^3} = \sqrt[3]{3^3(x^2)^3y^3} \tag{21}$$
$$= \boxed{3x^2y} \tag{22}$$

(f) Rewrite with rational exponent(s): $\dfrac{\sqrt[3]{y^2}+\sqrt{x^5}}{y-2}$

Solution:

$$\frac{\sqrt[3]{y^2}+\sqrt{x^5}}{y-2} = \frac{(y^2)^{\frac{1}{3}}+(x^5)^{\frac{1}{2}}}{y-2} \tag{23}$$
$$= \boxed{\frac{y^{\frac{2}{3}}+x^{\frac{5}{2}}}{y-2}} \tag{24}$$

4. The following are unrelated: (10 pts)

(a) Subtract: $\dfrac{1}{x^2+x}-\dfrac{1}{x^2+3x+2}$

Solution:

$$\frac{1}{x^2+x}-\frac{1}{x^2+3x+2} = \frac{1}{x(x+1)}-\frac{1}{(x+2)(x+1)} \tag{25}$$
$$= \frac{(x+2)}{x(x+1)(x+2)}-\frac{x}{x(x+2)(x+1)} \tag{26}$$
$$= \frac{x+2-x}{x(x+1)(x+2)} \tag{27}$$
$$= \boxed{\frac{2}{x(x+1)(x+2)}} \tag{28}$$

(b) Simplify the complex fraction: $\dfrac{\frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x-1}}$

Solution:

$$\frac{\frac{1}{x} - \frac{1}{x^2}}{1 + \frac{1}{x-1}} = \frac{\left(\frac{x}{x^2} - \frac{1}{x^2}\right)}{\left(\frac{x-1}{x-1} + \frac{1}{x-1}\right)} \tag{29}$$

$$= \frac{\left(\frac{x-1}{x^2}\right)}{\left(\frac{x-1+1}{x-1}\right)} \tag{30}$$

$$= \frac{\left(\frac{x-1}{x^2}\right)}{\left(\frac{x}{x-1}\right)} \tag{31}$$

$$= \frac{x-1}{x^2} \cdot \frac{x-1}{x} \tag{32}$$

$$= \boxed{\frac{(x-1)^2}{x^3}} \tag{33}$$

5. For what value of c is the number $x = 9$ a solution of the equation $\sqrt{x} + cx = 4c$? (5 pts)

Solution:

Since $x = 9$ is a solution to the equation, we can plug in the value and solve for c.

$$\sqrt{x} + cx = 4c \tag{34}$$

$$\sqrt{9} + c(9) = 4c \tag{35}$$

$$3 + 9c = 4c \tag{36}$$

$$3 = -5c \tag{37}$$

$$-\frac{3}{5} = c \tag{38}$$

So $\boxed{c = -\frac{3}{5}}$ is the value such that $x = 9$ is a solution to the equation.

6. Solve each of the following equations utilizing the properties of equations to justify your answers: (20 pts)

(a) $x^3 = 8x^2 - 16x$

Solution:

First, we rearrange to get a zero on one side:

$x^3 - 8x^2 + 16x = 0$

Then we factor:

$$x^3 - 8x^2 + 16x = 0 \tag{39}$$

$$x(x^2 - 8x + 16) = 0 \tag{40}$$

$$x(x-4)(x-4) = 0 \tag{41}$$

Then, using the multiplicative property of zero, the solutions are $\boxed{x = 0}$ and $\boxed{x = 4}$.

(b) $|3x-1|=4$

Solution:

$$|3x-1| = 4 \tag{42}$$
$$3x-1 = \pm 4 \tag{43}$$

This leads to 2 solutions:

$$3x-1 = 4 \tag{44}$$
$$3x = 5 \tag{45}$$
$$x = \frac{5}{3} \tag{46}$$

and

$$3x-1 = -4 \tag{47}$$
$$3x = -3 \tag{48}$$
$$x = -1 \tag{49}$$

Hence, the solutions are $\boxed{x = \frac{5}{3}}$ and $\boxed{x=-1}$.

(c) $\sqrt{-1-x} - x = 3$

Solution:

We start by isolating the square root and we get: $\sqrt{-1-x} = x+3$

Then we square both sides and proceed to solve the equation:

$$\left(\sqrt{-1-x}\right)^2 = (x+3)^2 \tag{50}$$
$$-1-x = (x+3)^2 \tag{51}$$
$$-1-x = x^2+6x+9 \tag{52}$$
$$x^2+7x+10 = 0 \tag{53}$$
$$(x+2)(x+5) = 0 \tag{54}$$

Using the multiplicative property of zero, this gives us $x=-2$ and $x=-5$. However, plugging these into the original equation we notice that only $x=-2$ satisfies the equation. Hence the solution is $\boxed{x=-2}$.

(d) Solve for h: $S = 2l^2 + 4lh$

Solution:

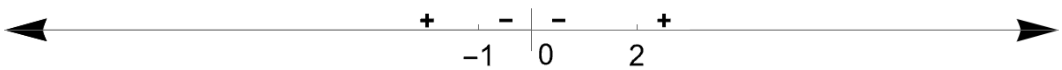
$$S = 2l^2 + 4lh \tag{55}$$
$$S - 2l^2 = 4lh \tag{56}$$
$$h = \boxed{\frac{S-2l^2}{4l}} \tag{57}$$

7. Solve the following inequalities. To receive credit, be sure to justify answers with a real number line or sign chart when appropriate. Express all answers in interval notation. (12 pts)

(a) $x^2(x+1)(x-2) \geq 0$

Solution:

We get three values that make the left side zero: $x = 0$, $x = -1$ and $x = 2$. Placing these on a number line and picking test values we obtain the following chart:



Notice that $x = 0$ is a solution. Hence the solution in interval notation is $\boxed{(\infty, -1] \cup 0 \cup [2, \infty]}$.

(b) $|x-1| \leq 4$

Solution:

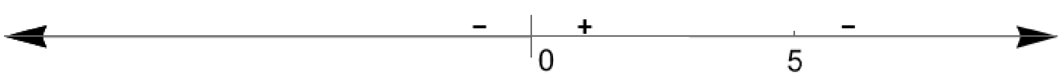
$$|x-1| \leq 4 \tag{58}$$
$$-4 \leq x-1 \leq 4 \tag{59}$$
$$-3 \leq x \leq 5 \tag{60}$$

Hence the solution in interval notation is $\boxed{[-3, 5]}$.

(c) $\dfrac{-2x}{x-5} \geq 0$

Solution:

For this rational expression, we notice that $x = 0$ makes the numerator zero, and $x = 5$ makes the denominator zero. Placing these on a number line and picking test values we obtain the following chart:



Notice that $x = 5$ is not an allowed value for this expression. Hence the solution in interval notation is $\boxed{[0, 5)}$.

8. The height of an object can be calculated for any time t from 0 seconds to 3 seconds by evaluating the expression: $-16t^2 + 48t$ for a specific time t. The height is measured in feet. (8 pts)

   (a) Express all times for which it's possible to calculate a height. Give your answer in interval notation. (3 pts)

   Solution:

   $\boxed{[0, 3]}$

   (b) What is the height of the object at time $t = \dfrac{1}{2}$ seconds?

   Solution:

   We obtain the height by plugging in $t = \frac{1}{2}$ in the given expression for height:

$$
\begin{aligned}
-16\left(\frac{1}{2}\right)^2 + 48\left(\frac{1}{2}\right) &= -16\left(\frac{1}{4}\right) + 24 && (61)\\
&= -4 + 24 && (62)\\
&= \boxed{20 \text{ feet}} && (63)
\end{aligned}
$$

END OF EXAM