Write your name and section number below. This exam has 5 problems and is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use one handwritten 3 x 5 inch notecard (front and back) on this exam. You are NOT allowed to use any other notes, books, calculators, or electronic devices.

After you finish the exam, go to the designated area of the room to scan and upload your exam to Gradescope. **Please be sure to match your work with the corresponding problem.** Do not leave the room until you verify that your exam has been correctly uploaded.

Name:

Instructor/Section (Dougherty-001, Mitchell-002, Becker-003):

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1. (31 points) For each of the following, **provide a short proof or justification**.

   (a) (8 points) If A and B are symmetric matrices of the same size, is AB symmetric?

   (b) (7 points) Let A be the matrix with permuted LU factorization given by the matrices

   $$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 4 & -2 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

   What is the determinant of A?

   (c) (8 points) Recall that A and B are similar, written $A \sim B$, if there exists an invertible matrix S with $B = S^{-1}AS$. If $A \sim B$, is $det(A) = det(B)$?

   (d) (8 points) Define the trace of a square, $n \times n$ matrix A, to be $tr(A) = \sum_{i=1}^{n} a_{ii}$. For $n \times n$ matrices A and B, show that $tr(AB) = tr(BA)$.

2. (14 points)

   Let A be the matrix with permuted LU factorization given by the matrices

   $$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 4 & 2 & 4 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

   and $\vec{b} = \begin{bmatrix} 0 \\ 6 \\ 5 \\ -2 \end{bmatrix}$

   Find the solution $\vec{x}$ to $A\vec{x} = \vec{b}$. *To help check that your final answer is correct, you should have $x_1 + x_2 + x_3 + x_4 = 1$.*

3. (15 points) Let $A = \begin{pmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & -3 \end{pmatrix}$. Find A^{-1}. Also **verify that you have indeed found** A^{-1}. *Hint: To help check your work, A^{-1} should only involve integers, no decimals/fractions.*

4. (20 points) For which values of b and c does the system $x_1 + x_2 + bx_3 = 1$, $2x_1 + 3x_2 - x_3 = -2$, and $3x_1 + 4x_2 + x_3 = c$ have (a) no solution? (b) exactly one solution? (c) infinitely many solutions? (Note: you do not have to find the solutions, just find the values of b and c and explain your work.)

5. (20 points) Let $A = \begin{pmatrix} 2 & 1 & 4 & 11 \\ -2 & 0 & -2 & -8 \\ 4 & 3 & 10 & 25 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$. Find the general solution to $A\vec{x} = \vec{b}$.