Write your name below. This exam is worth 150 points. You must show all your work to receive credit on each problem and must fully simplify your answers unless otherwise instructed. You are allowed to use one page of notes. You are not allowed to use a calculator or any computational software.

Name:

---

1. (15 points) An out of control rocket is traveling along the space curve given by $\mathbf{r}(t) = (5t + t^2)\mathbf{i} + 2\cos\left(\frac{\pi t}{3}\right)\mathbf{j} + \left(8 + 2\sin\left(\frac{\pi t}{3}\right)\right)\mathbf{k}$.
   (a) (6 points) Find the velocity of the rocket when $t = 5$.
   (b) (9 points) Assuming that the rocket exhaust is ejected in the opposite direction of the rocket's velocity, where does the exhaust ejected at $t = 5$ intersect the xy-plane?

2. (20 points) The following two problems are unrelated.

   (a) (6 points) Find the following limit or prove that it does not exist:

   $$\lim_{(x,y)\to(0,1)} \frac{x - xy}{x^2 - (y-1)^2}$$

   (b) (14 points) Find the degree two Taylor approximation of $f(x,t) = \sin(3x - 4t)$ near the point $(0,0)$.

3. (20 points) Let D be the triangular region in the xy-plane with vertices $(0,0)$, $(2,1)$, $(1,2)$ and f be the function $f(x,y) = x^2 + 2xy + y^2$ and T be the coordinate transformation given by

$$u(x,y) = x + y$$
$$v(x,y) = 2x - y$$

   (a) (6 points) Set up but do not evaluate the integral of f over D in the xy-plane.

   (b) (6 points) Sketch the image of D in the uv-plane, labeling all relevant points.

   (c) (8 points) Set up but do not evaluate the integral of f over the image of D in the uv-plane.

4. (20 points) The Pteranodon enclosure for the local Cretaceous Garden theme park consists of the volume bounded on top by a sphere centered on the origin and below by the upper half of a cone, again centered at the origin. The intersection of these surfaces occurs at $r = \frac{5\sqrt{3}}{2}$ and $z = \frac{5}{2}$ in cylindrical coordinates. As Pteranodons prefer to fly high, the density of Pteranodons is given by $f(x, y, z) = 10z - x^2 - y^2$ in rectangular coordinates.

   (a) (8 points) Set up but do not evaluate the integral of f over the enclosure in cylindrical coordinates.

   (b) (12 points) Set up but do not evaluate the integral of f over the enclosure in rectangular coordinates.

5. (22 points) Given the following system of linear equations

$$\begin{aligned} 2x_1 - 4x_2 + 6x_4 &= 2 \\ -x_1 + 2x_2 + x_3 - 2x_4 &= -2 \\ 2x_1 - 4x_2 - x_3 + 5x_4 &= 3 \end{aligned}$$

   (a) (10 points) Find the RREF of the augmented matrix for the system.

   (b) (4 points) Identify the independent and dependent variables of the system.

   (c) (8 points) Find the general solution for the system.

6. (15 points) Let $A = \begin{pmatrix} 1 & a \\ a & 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. If possible, find values of a where the equation $A\mathbf{x} = \mathbf{b}$ has

   (a) No solutions.

   (b) One unique solution.

   (c) Infinite solutions.

   If no such value of a exists, write 'Not possible.'

7. (15 points) Let A be the matrix given by the product:

$$A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 1 & 3 \end{pmatrix}$$

Use the properties of matrix inverses to find A^{-1} without calculating A.

8. (23 points) Given the points $(-2,-2)$, $(-1,1)$, $(0,2)$, and $(0,3)$

   (a) (16 points) Find the least squares best fit line through the points.

   (b) (7 points) Calculate the error vector.