INSTRUCTIONS: **Simplify** and **box** all your answers. Write neatly and **show all work**. A correct answer with incorrect or no supporting work may receive no credit. Books, notes, electronic devices (such as calculator or other unauthorized electronic resources), help from another person, are not permitted during the exam. **Give all answers in exact form.**

Potentially useful formulas:

1. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
2. Circle: $(x-h)^2 + (y-k)^2 = r^2$
3. Arc length: $s = r\theta$
4. $\sin(a-b) = \sin a \cos b - \sin b \cos a$
5. $\cos(a-b) = \cos a \cos b + \sin a \sin b$
6. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
7. $\cos(2\theta) = 2\cos^2\theta - 1$
8. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$
9. $\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$
10. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
11. Area of a sector: $A = \frac{1}{2}r^2\theta$
12. $\sin(a+b) = \sin a \cos b + \sin b \cos a$
13. $\cos(a+b) = \cos a \cos b - \sin a \sin b$
14. $\sin(2\theta) = 2\sin\theta\cos\theta$
15. $\cos(2\theta) = 1 - 2\sin^2\theta$
16. $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$
17. $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$

NOTE: TEAR THIS PAGE OFF AND USE (FRONT AND BACK) AS SCRATCH PAPER.

i. DO NOT START UNTIL INSTRUCTED BY A PROCTOR.

ii. THE EXAM IS ON BOTH SIDES OF EACH EXAM PAGE

iii. WRITE YOUR NAME ON THE FIRST EXAM PAGE.

iv. WHEN YOU FINISH (IF BEFORE THE EXAM END TIME) PLEASE QUIETLY COLLECT YOUR THINGS AND MOVE TO THE EXAM SUBMISSION AREA. SUBMIT YOUR EXAM TO GRADESCOPE AND TURN IN THE PAPER COPY TO THE PROCTOR.

Scratch Paper

Name: _____

1. The following are unrelated. (6 pts)

   (a) Perform the indicated operations: $e^0 + \frac{5}{12} - \frac{11}{18}$

   (b) Evaluate the expression: $-2\sqrt{32} + \sqrt{50}$

2. Rewrite each of the following without absolute value symbol (4 pts):

   (a) $|x - 2|$ where $x > 2$

   (b) $|1 - \sqrt{2}|$

3. Let a and b be real numbers such that $a \geq 0$ and $b < 0$. Determine whether the following expression is positive, negative, or the sign cannot be determined (2 pts):

$$2ab - 15b^2 3^b$$

4. The following are unrelated. (8 pts)

   (a) Rewrite the expression with positive exponents and combine: $(a-1)^{-3}(a-1)^{-14}$

   (b) Perform the indicated operations: $x^4 x^{\frac{3}{2}} + \dfrac{3x^{-2}}{4\left(x^3\right)^{-\frac{5}{6}}}$

5. The following are unrelated. (8 pts)

   (a) Find the sum, difference, product as indicated: $(2x-4)^2 - (x+2) + 2(1-3x)$

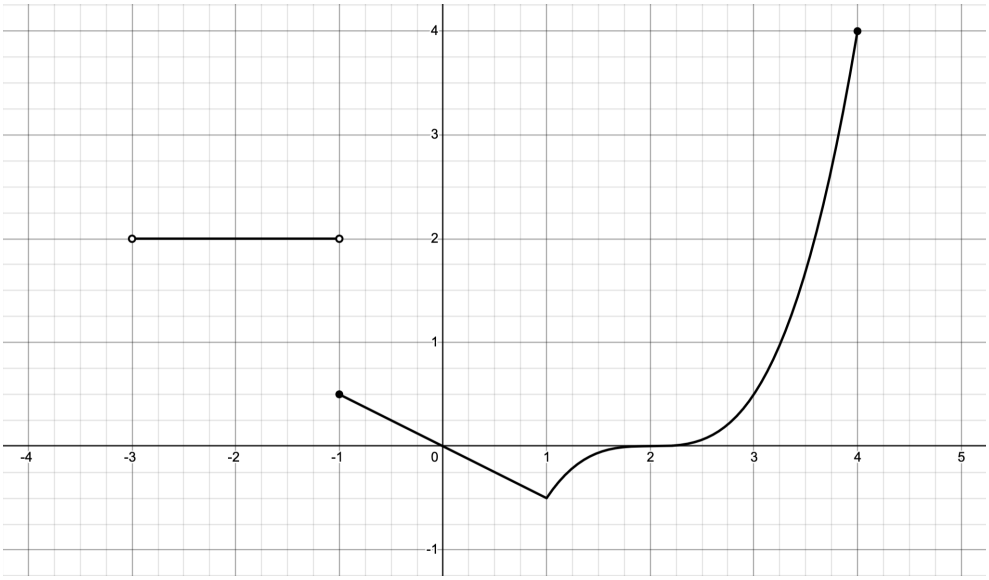
   (b) Multiply and simplify: $\left(5 - \sqrt{1+x}\right)^2$

6. The following are unrelated. (9 pts)

(a) Perform the division and simplify: $\dfrac{\frac{3x^2-27}{x^2-6x+9}}{\frac{4x^2-16x}{2x-8}}$

(b) Simplify the following: $\ln\left(e^3\right) + \log_3(3x) - \log_3(x) + \log_2(16)$ (Your answer should have no logarithms)

7. Use the graph of $y = f(x)$ below to answer the following (10 pts):



(a) Identify the domain of $f(x)$.

(b) Identify the range of $f(x)$.

(c) Solve the inequality $f(x) \geq \frac{1}{2}$. Give answer in interval notation.

(d) If $h(x) = e^{2x}$ find $h(f(-2))$.

(e) If $g(x) = 2^x - 1$ solve the equation $g(x) = f(x)$.

8. Use long division to find the **quotient** and **remainder** when $x^4 - 3x^3 - x^2$ is divided by $x^3 - 2x^2 + x$. (4 pts)

9. Solve the following equations for the indicated variable. If there are no solutions, write **no solutions**. (8 pts)

   (a) Solve for y: $\frac{3}{5}y - 1 = 2 + \frac{13}{10}y$

   (b) Solve for x: $2x(x - 6) = -32$

10. Solve for r when a is a constant and $a > 3$ (4 pts): $\log(3) = \log(2a) - \log(2 + r)$

11. If the recommended dosage of an adult drug is D (in mg), then to determine the appropriate dosage c for a child of age a (in years), pharmacists use the linear function $c = 0.12D(a + 1)$. Suppose the dosage for an adult is 100 mg. (6 pts)

    (a) What is the slope of the line (your answer should not include any variables)?

    (b) What are the units of the slope?

    (c) Find the average rate of change of the child dosage c from age 3 to age 9.

12. Consider the functions: $h(x) = \sin(x)$ and $k(x) = \dfrac{3}{x}$. (8 pts)

    (a) Fill in the blanks for the function $k(x)$: $k(x) \to$ ____ as $x \to -\infty$ and $k(x) \to$ ____ as $x \to \infty$.

    (b) Find $(h \circ k)(x)$.

    (c) Find the domain of $(h \circ k)(x)$. Give your answer in interval notation.

13. For the rational function $r(x) = \dfrac{x-3}{3x^2-6x-9}$ answer the following (11 pts):

(a) Find the domain of $r(x)$.

(b) Find the x-coordinate of any hole(s). If there are no hole(s) write NONE.

(c) Find the y-coordinate of any hole(s). If there are no hole(s) write NONE.

(d) Identify the horizontal or slant asymptote of $r(x)$. If there is no horizontal or slant asymptote write NONE.

(e) Find all vertical asymptote(s). If there are none write NONE.

14. Answer the following for the polynomial function $y = P(x)$ with the following properties. (8 pts)

i. The graph of $y = P(x)$ touches (does not cross) the x-intercept $(0, 0)$.

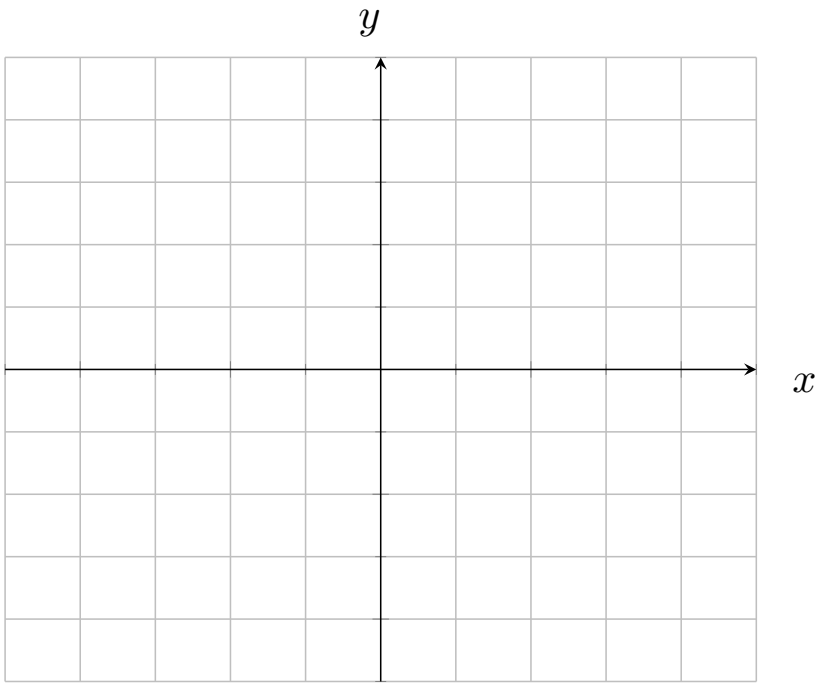
ii. The graph of $y = P(x)$ crosses the x-intercept $(-2, 0)$.

iii. The graph of $y = P(x)$ satisfies: $P(x) \to -\infty$ as $x \to -\infty$.

iv. $y = P(x)$ is an even function.

v. $y = P(x)$ has one, and only one, additional x-intercept to those listed above.

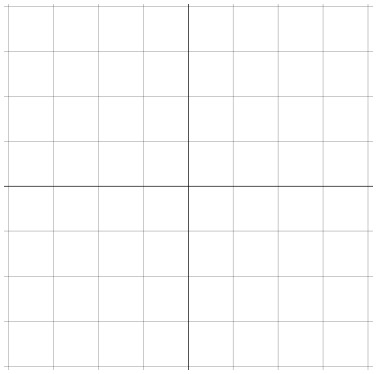
(a) Sketch the graph of $y = P(x)$ that satisfies all the given properties. **Label all intercepts.**



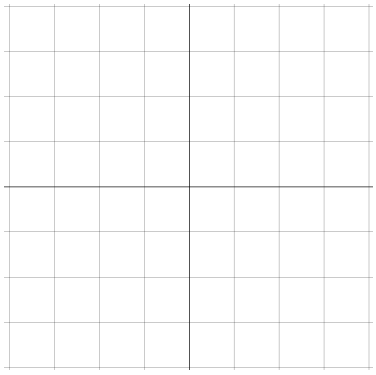
(b) Find the equation of a degree 4 polynomial function for $P(x)$ that satisfies all of the given properties.

15. Sketch the graph of the following functions. **Label all intercepts and asymptotes as appropriate.** (15 pts)

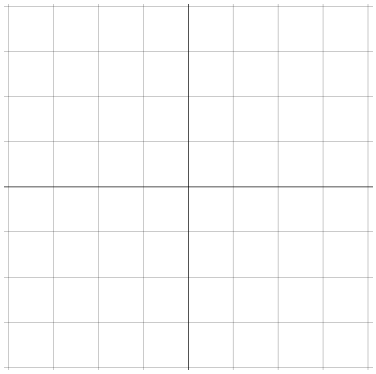
(a) $g(x) = -|x-1|$



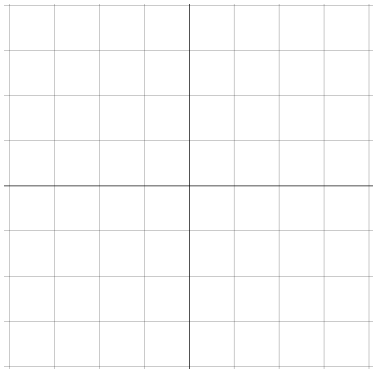
(b) $n(x) = \begin{cases} e^x & \text{if} \quad x \leq 0 \\ -2x+1 & \text{if} \quad 0 < x < 2 \\ \pi & \text{if} \quad x = 2 \end{cases}$



(c) $r(x) = \tan(x)$ on the restricted domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



(d) $j(x) = \tan^{-1}(x)$



(e) For $j(x)$ in part (d) fill in the blank: $j(x) \to$ ____ as $x \to \infty$.

16. Find the exact value for each (do not attempt to find decimal approximations): (12 pts)

(a) $\cos\left(\frac{\pi}{2}\right)$

(b) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\tan\left(-135^\circ\right)$

(d) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

17. Find the exact value for the given expression (do not attempt to find a decimal approximation): (4 pts)

$$\sin\left(-\frac{3\pi}{8}\right)$$

18. Simplify the expression: $\dfrac{\cos(\theta)\sec(\theta) - \sin^2(\theta)}{2\cos(\theta)}$. (4 pts)

19. Find all solutions to the following equations: (8 pts)

(a) $2\cos^2(\theta) = -\cos(\theta)$

(b) $\cos(2x)\cos(x) + \sin(2x)\sin(x) = \dfrac{1}{2}$

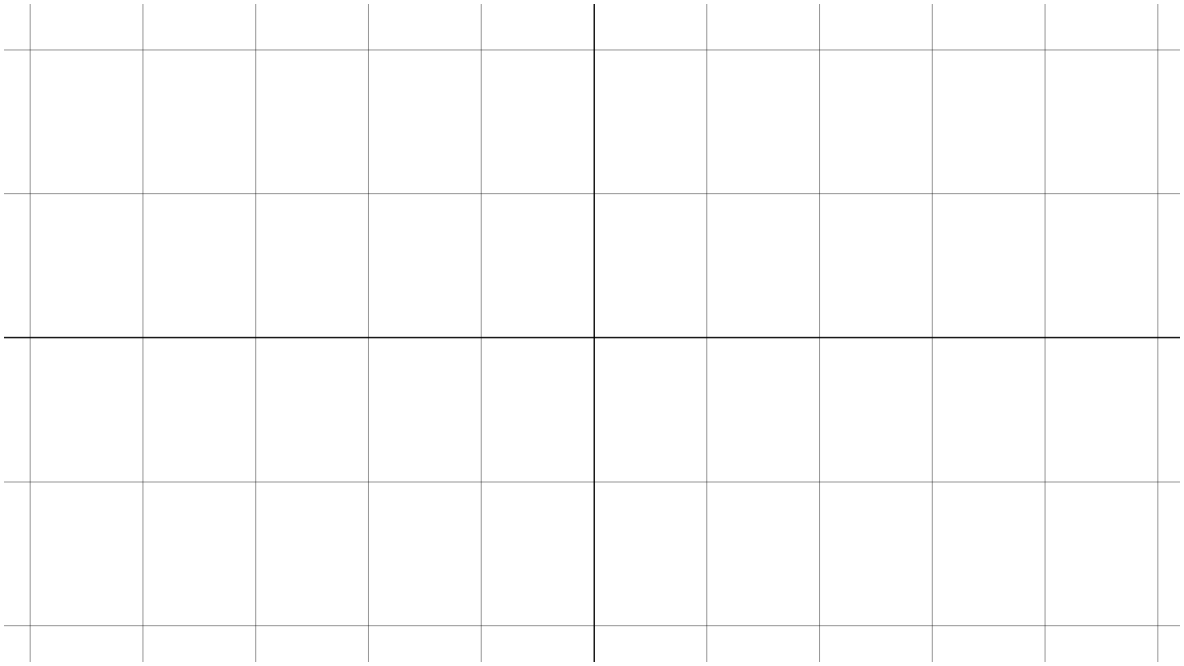
20. For $m(x) = \frac{3}{2}\sin(4x)$ (7 pts)

(a) Identify the amplitude.

(b) Identify the period.

(c) Identify the phase shift.

(d) Sketch one cycle of the graph of $m(x)$. **Label** at least two values on the x-axis and clearly identify the amplitude.



21. Two surveyors are standing in a flat field with a 50 foot tall tree between them. The surveyors need to know how far apart they are standing from each other. One surveyor measures the angle of elevation from the ground where they stand to the top of the tree to be $30°$ while the other measures the angle of elevation from the ground where they stand to the top of the tree to be $45°$. How far apart, in feet, are the two surveyors? Give the exact answer, do not attempt to approximate with decimal values. (4 pts)

END OF FINAL EXAM