On the front of your bluebook, write (1) **your name**, (2) **Exam 3**, (3) **APPM 3570/STAT 3100**. Correct answers with no sup-
porting work may receive little or no credit. Books, notes and electronic devices of any kind are not allowed. Your exam should be
uploaded to Gradescope in a PDF format (Recommended: **Genius Scan**, **Scannable** or **CamScanner** for iOS/Android). **Show
all work, justify your answers. <u>Do all problems.</u>** Students are required to re-write the **honor code statement** in the box
below on the **first page** of their exam submission and **sign and date it**:

1. (30pts) There are 3 unrelated parts to this question. Justify your answers.

    (a) (10pts) Suppose we roll a pair of fair, six-sided dice. If X is the number of dice that come up 1, and Y is the number
    of dice that come up 4, 5 or 6, find $P(Y = 1 | X = 0)$.

    (b) (10pts) Suppose 10 balls are put into 5 urns, with each ball independently being put in urn i with probability p_i, where
    $\sum_{i=1}^{5} p_i = 1$. What is the expected number of urns that contains at least one ball?

    (c) (10pts) Suppose the random variables X, Y and Z have the means $\mu_X = 2$, $\mu_Y = -3$ and $\mu_Z = 4$, the variances
    $\sigma_X^2 = 1$, $\sigma_Y^2 = 5$ and $\sigma_Z^2 = 2$ and the covariances $\text{cov}(X, Y) = -2$, $\text{cov}(X, Z) = -1$ and $\text{cov}(Y, Z) = 1$. If we define
    $U = X - Y$ and $V = X + Z$, find $\text{cov}(U, V)$.

2. (40pts) A nut company markets cans of deluxe mixed nuts containing *almonds*, *cashews*, and *peanuts*. Suppose the net
weight of each can is 1 lb, but the weight contribution of each type of nut is random. Let X be the weight of almonds in
a selected can and Y the weight of cashews. The joint pdf of (X, Y) is given to be

$$f(x, y) = \begin{cases} 24xy, & \text{for } 0 < x < 1,\ 0 < y < 1 \text{ and } x + y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

    (a) (10pts) Set up, but *do not solve*, an integral (or integrals) to find the probability that the *peanuts* make up at
    least 60% of the can.

    (b) (10pts) Find $f_X(X)$, the marginal pdf for the weight of almonds, X, and find the expectation $E[X]$.
    (Be sure to define the pdf for all values of $\mathbb{R}$.)

    (c) (10pts) Suppose 1 lb of almonds fills up 2.8 cups, 1 lb of cashews fills 3.5 cups and 1 lb of peanuts fills 2.7 cups.
    Find the expected volume of the contents of a randomly selected can of deluxe nuts (in terms of cups). (*Hint:* No
    further calculations are needed to find $E[Y]$.)

    (d) (10pts) Find the conditional pdf Y given that $X = 0.75$ and the conditional expectation of $Y | X = 0.75$.

3. (30pts) Let X and Y be independent and identically distributed *Exponential* random variables each with parameter $\lambda = 1$.

    (a) (10pts) Find the probability $P\left(\dfrac{Y}{X} < 2\right)$. Show all work.

    (b) (10pts) Let $U = X + Y$ and $V = \dfrac{X}{X + Y}$, find $f_{U,V}(u, v)$, the *joint probability density function* of (U, V).

    (c) (10pts) Find the marginal pdf of the random variable V, be sure to specify the domain. Are U and V independent?
    Why or why not?