1. (34 points) The following problems are unrelated.

   (a) Find the tangent line of $f(x) = \sqrt{x^5} + \dfrac{4x^8}{3ax^{2/3}}$ at $x = 1$. Write your final answer in point-slope form. Your answer will be in terms of the constant $a$.

   (b) Evaluate $\displaystyle\int_1^e \frac{1}{x(\ln x)^2 + x}\,dx$.

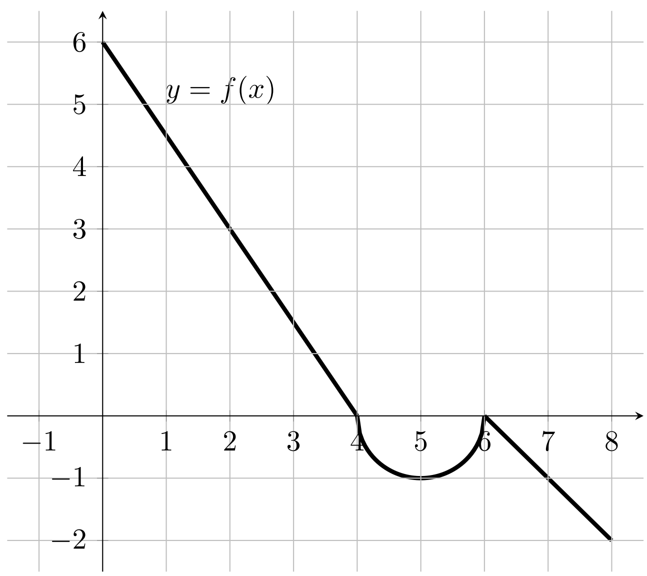
   (c) Evaluate $\displaystyle\int \frac{\sinh(\tan x + 4)}{\cos^2 x}\,dx$.

2. (18 points) Consider $g(x) = x^{1/(1-x)}$ as you answer both of the following.

   (a) Evaluate $\displaystyle\lim_{x \to 1^+} g(x)$.

   (b) Find $g'(x)$. (Find $g'(x)$ in terms of $x$, but please DO NOT simplify your answer further.)

3. (22 points) Consider the function $f(x)$ defined over $[0, 8]$ that is graphed below. It consists of two straight line segments and a semicircle.




   (a) Find the average value of $f$ over the interval $[0, 8]$.

   (b) Evaluate $\displaystyle\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$.

   (c) Let $g(x) = \displaystyle\int_0^x f(t)\,dt$. Find the tangent line of $y = g(x)$ at $x = 2$.

4. (21 points) The following two problems are not related.

   (a) Particle A moves along an axis. Its velocity on this axis is provided for specific times in the table below. Approximate the displacement of Particle A from $t = 0$ to $t = 12$ in two ways:

      i. Using the upper sum rule with $n = 6$ intervals of equal width.

| time (seconds) | 0 | 2 | 4 | 6 | 8 | 10 | 12 |
|---|---|---|---|---|---|---|---|
| velocity (meters per second) | 10 | 12 | 8 | 5 | 0 | 2 | 5 |

ii. Using the midpoint rule with $n = 3$ intervals of equal width.

(b) Suppose Particle B moves along a different axis, with an initial velocity of 20 m/s. If it comes to a complete stop after traveling 60 meters, what must its constant rate of acceleration have been?

5. (24 pts) Consider $r(x) = \dfrac{e^{2x}}{e^{2x} + 3}$.

   (a) Determine the horizontal asymptote(s) of $y = r(x)$, or show that there are none. Be sure to justify your answer with the appropriate limits.

   (b) Find $r'(x)$ and use this to show that $r$ is one-to-one.

   (c) Determine the formula for the inverse function, $r^{-1}(x)$. (Clearly label your final answer as $r^{-1}(x)$.)

   (d) Determine the domains and ranges of $r(x)$ and $r^{-1}(x)$.

6. (15 points) At 12pm, the population of a bacteria culture is 5,000. By 3pm, the population is 6,000. Assuming the growth of the population is proportional to the current size of the population, that is $\frac{dP}{dt} = kP$, what will the population be at 7pm? (We know you do not have a calculator. Leave your answer as an exact answer.)

7. (16 points) An island is 6 miles due north of its closest point along a straight shoreline. A cabin on the shoreline is 8 miles west of that point. Jane is currently on the island and is planning to go from the island to the cabin. Jane has access to a sailboat. Jane can run along the shoreline at a rate of 5 mph and sail through the water at a rate of 3 mph. Jane is not allowed to go east of the island or west of the cabin, and she can only sail in a straight line. Let $x$ represent the number of miles down the shoreline Jane will sail before running the rest of the way.

   Note: $15^2 = 225$.

   (a) Give the formula for a function for the total time it takes to go from the island to the cabin. This function should be in terms of $x$.

   (b) How far down the shoreline should Jane sail before running the rest of the way to **minimize** the time it takes to reach the cabin? (Be sure to justify that you have found the absolute minimum.)

   (c) How far down the shoreline should Jane sail before running the rest of the way to **maximize** the time it takes to reach the cabin? (Assume Jane must run and sail at the rates mentioned above. Be sure to justify that you have found the absolute maximum.)