APPM 1340
Final Exam
Fall 2024

Name	
Instructor Richard McNamara	**Section** 150

This exam is worth 150 points and has **8 problems**.

Make sure all of your work is written in the blank spaces provided. If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

Show all work and simplify your answers. Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

End-of-Exam Procedure

1. Go to the designated area to scan and upload your exam to Gradescope.
2. Verify that your exam has been correctly uploaded and all problems have been labeled.
3. Hand the physical copy of your exam to a proctor.
4. Have a proctor swipe your BuffOne card.

Potentially Useful Formulas

Sums and differences:

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

Double-angle formulas:

$\sin 2\theta = 2\sin\theta\cos\theta$
$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

1. (20 pts) The position function of a particle is given by $s(t) = t^{3/2} - 3t + 10$ on the interval $1 \leq t \leq 9$, where position is in meters and time is in seconds.

   (a) Determine the particle's velocity function $v(t)$. Include the correct unit of measurement.

   (b) Determine the total distance traveled by the particle on the interval $1 \leq t \leq 9$. Include the correct unit of measurement.

2. (17 pts) Consider the function $f(x) = \dfrac{(x-2)^2}{x-5}$.

(a) Identify all critical numbers of f on the interval $(-\infty, \infty)$.

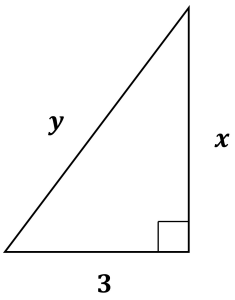
$$f(x) = \frac{(x-2)^2}{x-5}$$

(b) The Extreme Value Theorem indicates that $f(x)$ attains an absolute maximum value and an absolute minimum value on the interval $[0, 4]$. Identify the (x, y) coordinates of every point at which f attains one of those extreme values on that interval. Be sure to clearly indicate which point(s) correspond to the maximum value and which correspond to the minimum value.

(*Hint:* Use the Closed Interval Method.)

3. (19 pts) Find an equation of the tangent line to the curve $\sin(x - y) = \sin y - \cos x$ at the point $(\pi/4, \pi/4)$.

4. (16 pts) The following questions relate to the right triangle depicted here:



(a) Write an expression for y as a function of x.

(b) Find the linear approximation of $y(x)$ at $a = 4$.

(c) Suppose the value of x is measured to be 4 cm with a possible error in measurement of at most 0.25 cm. Use differentials to estimate the maximum error in computing the value of y based on the measured value of x. Include the correct unit of measurement.

5. (20 pts) Parts (a) and (b) are not related.

   (a) At $x = 2$, does the function $g(x) = \dfrac{x^2 - 5x + 6}{x^2 - 4x + 4}$ have a removable discontinuity, a vertical asymptote, or neither? Support your answer by evaluating the appropriate limit(s).

   (b) Find the equation of each horizontal asymptote of $y = h(x) = \dfrac{2x - 3x\sqrt{x} + 1}{2x^{3/2} + 3x - 1}$, if any exist. Support your answer by evaluating the appropriate limit(s).

   (Note: You may not use L'Hôpital's Rule or dominance of powers arguments to evaluate limits on this exam.)

6. (18 pts) Verify that the hypotheses of the Mean Value Theorem are satisfied for the function $u(x) = x - \cos^3 x$ on the interval $[0, 2\pi]$, and find all numbers c that satisfy the conclusion of that theorem.

7. (20 pts) A spotlight on the ground shines on a wall 25 feet away. If a man who is 6 feet tall walks from the wall toward the spotlight at a constant speed of 4 feet per second, how fast is the height of his shadow increasing when he is 10 feet from the light? Include the correct unit of measurement in your answer.

8. (20 pts) Consider the function $w(x)$, defined as follows:

$$w(x) = \begin{cases} 2\sqrt{x} & , \quad x < 1 \\ \\ x^2 - x + 2 & , \quad x \geq 1 \end{cases}$$

Is $w(x)$ differentiable at $x = 1$? Fully justify your answer using the limit definition of a derivative.

Note: $w(x)$ is continuous at $x = 1$; you do not have to prove its continuity.

END OF EXAM

Your Initials _____

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.