1. The following two problems are not related.

   (a) (12 pts) Evaluate $\int \sin x \cos x \, e^{\sin x}\, dx$. (*Hint:* Begin by making a substitution.)

   (b) (10 pts) Let $I = \int_{100}^{k} \frac{1}{(x-k)^3}\, dx$ where $k$ is a constant. Evaluate $I$.
   Specify the values of $k$ for which $I$ is convergent.

2. (22 pts) Determine whether the following expressions are convergent or divergent. If convergent, find the value the expression converges to. Be sure to fully justify your answers.

   (a) $a_n = n \sin\left(\frac{\pi}{n}\right)$  (b) $\sum_{n=1}^{\infty} \frac{1}{(\ln 2)^{n/2}}$  (c) $\sum_{n=2}^{\infty} \frac{6^{n-1}}{5^n (\ln n)^{10}}$

3. The following two problems are not related.

   (a) (8 pts) The geometric series $2 + \frac{8}{m} + \frac{32}{m^2} + \cdots$ has a sum of 5. What is the value of $m$?

   (b) (8 pts) A power series $\sum_{n=0}^{\infty} c_n (x+3)^n$ converges for $x = 1$ and diverges for $x = 20$. Find the minimum and maximum possible values for the radius of convergence.

4. (26 pts) The Maclaurin series for a function $f(x)$ is $\sum_{n=0}^{\infty} \frac{5^{n-1}}{n!} x^{n+1}$.

   (a) Approximate the value of $f\left(\frac{1}{5}\right)$ using $T_2(x)$, the 2nd order Taylor polynomial.

   (b) Use Taylor's Remainder Formula to find an error bound for the approximation found in part (a). Express your answer in terms of $e$. *Hint:* $f''(x) = e^{5x}(5x+2)$.

   (c) Find the sum of the Maclaurin series.

5. (28 pts) Let $\mathcal{R}$ be the region in the 1st quadrant bounded by the curve $y = 2\sqrt{1-x^2}$ and the positive $x$ and $y$ axes.

   (a) Sketch region $\mathcal{R}$. Label intercepts.

   (b) Set up (but <u>do not evaluate</u>) integrals to find the following quantities. Calculate any derivatives but otherwise integrands may be left unsimplified.

      i. Volume of the solid generated by rotating $\mathcal{R}$ about the $x$-axis using the Disk/Washer method.

      ii. Volume of the solid generated by rotating $\mathcal{R}$ about the $x$-axis using the Shell method.

      iii. Length of the curve in the 1st quadrant.

6. The following two problems are not related.

   (a) (6 pts) A parametric representation for the curve $x^2 + \frac{y^2}{4} = 1$ in the 1st quadrant can be found by letting $x = \cos(\ln t)$. Find the corresponding equation for $y$ in terms of $t$ for $\alpha \leq t \leq \beta$.

   (b) (14 pts) Consider the curve $x = \ln(t+3)$, $y = (t+3)^2$.

      i. Use the parametric formula to find $\frac{d^2y}{dx^2}$ at $x = \ln 2$.

      ii. Find a Cartesian representation for the curve. Fully simplify and write your answer in the form $y = f(x)$.

7. (16 pts) Consider the polar curve $r = \cos(\theta/2)$.

   (a) Set up (but do not evaluate) an integral to find the length of the curve.

   (b) The curve contains two inner loops. Evaluate an integral to find the area inside one inner loop.